Initial-boundary value problems in linear viscoelasticity using Wiener-Hopf methods

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Introduction

The present work attempts to study wave propagation phenomena in *linear viscoelasticity theory* using formulations as initial-boundary value problems. Common solution schemes from the wide range of literature available on the subject are used and some aspects that might be new are joined.

In the first Chapter the origins of the equations of motion related to the particular linear hereditary stress-strain law underlying all considerations are provided. We note that the equations describe both solid- and fluid-like behaviour of the continuum. From the isotropic mechanical model we deduce the counterpart of the elastodynamic Lamé equations.

In Chapter 2 we investigate the scalar Cauchy problem. The equation of motion is transferred to the most general distributional setting in which we solved the problem explictly by means of the fundamental resolvent. Using Laplace transform techniques, we restricted ourselves to displacement distributions of maximal exponential growth.

In the third Chapter scalar initial-boundary value problems are brought into the weak setting where Dirichlet and Neumann boundary conditions are incorporated naturally. The explicit solutions of maximal exponential growth are obtained by means of the related Green function depending on the Laplace parameter. Eventually, we show that the solutions obtained are of finite energy.

In Chapter 4 we consider first some canonical vectorial problems on the half-space and then some related half-plane crack problems. The matrix symbol functions of the related boundary operators are again presented in the Laplace-transformed time domain. Then the equivalence of the crack problems and the corresponding Wiener-Hopf operators is deduced. Finally, we do the explicit factorization procedure of the Wiener-Hopf operators for two particular examples.

In Appendix A the properties of the material functions related to the mechanical model are discussed. Different material qualities are hence distinguished. As for Appendix B, we collected some operator-theoretic facts on the Laplace operator. Eventually, in Appendix C some basic results from the Sobolev space theory are provided. In Appendix D, we recall some facts on the Wiener-Hopf factorization.

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