Euclidean Sequences

Uwe Kraeft

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Berichte aus der Mathematik

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Preface

This book gives an arbitrary choice of some Euclidean Sequences. These are defined here as finite or infinite sequences of rational numbers whose members can be calculated by an **algorithm**. By this, no Euclidean Sequences are f.e. statistical sequences found by a chance generator, in which the „next" member is undetermined, and ambiguous sequences.

In chronological order with a former book on Pythagorean Triples this second text is written for all who are interested in basic mathematics. Especially it may be of interest for the student who begins to study the elements of number theory and history of mathematics. The book is dedicated to Euclid because his famous Elements and other works show an important progress in number theory. For a part Euclid has probably collected results of older philosophers and mathematicians. But there is certainly an own contribution. He had the luck that his books were copied many centuries and so preserved until now.

In this book are treated especially Euclidean Sequences of natural numbers.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in May 2000 Uwe Kraeft

Symbols

$\Rightarrow$	by this follows
$\in$	is element of (is contained in)
$\notin$	is no element of (isn't contained in)
$\subset$	is subset of (all elements are contained in)
$\cup, \cap$	union and intersection of sets
$\bigcap_{i=1}^{n} P_i$	$P_1 \cap P_2 \cap \dots \cap P_n$
$\varnothing$	the empty set
a, A, α ...	in this text mainly natural numbers 1, 2, 3, ...
N	set of natural numbers 1, 2, 3, ...
N^0	$N \cup \{0\}$
P^1	$P \cup \{1\}$
$\{-N\}$	$=\{-n; n \in N\}$ set of negative integers -1, -2, -3, ...
Z	$=N \cup \{-N\} \cup \{0\}$ set of integers
Q^+	set of positive rational numbers a/b, $N \subset Q^+$
R	set of real number algorithms
=	equal (not between rational and irrational numbers)
$\neq$	not equal
$\equiv$	congruent
	($a \equiv b \pmod{c}$ a congruent b modulo c means: $(a-b)/c \in N$)
$<$	less
$>$	greater
+,-	plus, minus
ab, a*b	a times b
$a/b = \frac{a}{b}$	a divided by b
(a,b,c)=1	greatest common divisor (factor) gcd$\in$N of a,b,c is 1
$\{a_i\}$	$\{a_1, a_2, \dots, a_n\}$ sequence of numbers
$\sum_{i=1}^{n} a_i$	$=a_1+a_2+ \dots +a_n$
$\binom{n}{m}$	$=\frac{n!}{m!(n-m)!}$ binomialcoefficients

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