

**Elektrotechnik**

Ömer Faruk Yildiz

**Functional Via Structures in  
Passive Microwave Components  
on Multilayer Ceramic Substrates**

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# **Functional Via Structures in Passive Microwave Components on Multilayer Ceramic Substrates**

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**Ömer Faruk Yildiz**

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# Summary

Low temperature cofired ceramic (LTCC) is a technology that distinguishes itself from conventional technologies such as printed circuit boards (PCBs) based on flame retardant (FR-4) or polytetrafluoroethylene (PTFE) substrate materials by means of higher permittivities, lower dielectric losses, and improved thermal performance. Its manufacturing process allows for the preparation of individual sheets and subsequent stacking, lamination, and finally firing into multilayer substrates with high dimensional accuracy. The ability to embed active or passive components between layers and form cavities enables a high degree of integration, thus making LTCC a viable packaging solution for a variety of electronic systems.

To this end, vertical interconnect accesses (vias), which are typically used to electrically connect traces or planes located on different layers, are deployed as functional elements for the design of vertically integrated microwave components. Functional via structures do not support transverse electromagnetic (TEM) modes but effectively behave like quasi-transmission lines up to 40 GHz for most practical purposes if designed properly. In this work, vertical integration is applied to a variety of passive microwave components including low- and band-pass filters, 90° and 180° hybrid couplers, and N-way Wilkinson power dividers. The analysis for each component is comprised of the delineation of the design procedure, full-wave simulations, and lastly measurements up to 50 GHz of the manufactured prototype. The study is completed by examining the limiting case of via arrays, where many signal vias are in close proximity to each other, which gives insight into maximum coupling and crosstalk behavior.

The utilization of the vertical dimension essentially grants circuit designers an additional degree of freedom not offered by planar microstrip or stripline technology. Furthermore, the reduced footprint and smaller form factor due to three-dimensional (3-D) integration implies shorter electrical lengths and enhanced performance.



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# Acronyms

2-D	two-dimensional
2.5-D	two-and-a-half-dimensional
3-D	three-dimensional
5G	fifth generation
AFSIW	air-filled substrate integrated waveguide
BEM	boundary element method
BGA	ball grid array
CIM	contour integral method
CPU	central processing unit
CPW	coplanar waveguide
CTLE	continuous-time linear equalizer
DoE	design of experiments
DUT	device under test
ECal	electronic calibration module
EMC	electromagnetic compatibility
EMI	electromagnetic interference
FEM	finite element method
FEXT	far-end crosstalk
FIT	finite integration technique
FR-4	flame retardant
GPU	graphics processing unit
GQ	Gaussian quadrature
GSG	ground-signal-ground
HTCC	high temperature cofired ceramic

## Acronyms

IC	integrated circuit
IF	intermediate frequency
IHF	Institut für Hochfrequenztechnik
IL	insertion loss
LCP	liquid crystal polymer
LGA	land grid array
LNA	low noise amplifier
LO	local oscillator
LSI	large-scale integration
LTCC	low temperature cofired ceramic
MC	Monte Carlo
MCM	multi-chip module
MEMS	microelectromechanical system
MMIC	monolithic microwave integrated circuit
NEXT	near-end crosstalk
PBV	physics-based via
PC	personal computer
PCB	printed circuit board
PCE	polynomial chaos expansion
PDF	probability density function
PGA	pin grid array
PI	power integrity
PSFEXT	power sum of far-end crosstalk
PSNEXT	power sum of near-end crosstalk
PTFE	polytetrafluoroethylene
PTH	plated through-hole
RAM	random access memory
RF	radio frequency
RFIC	radio frequency integrated circuit
RL	return loss
RPL	recessed probe launch
RSM	response surface methodology
SI	signal integrity
SIC	stacked integrated circuit

SiP	system in package
SIW	substrate integrated waveguide
SMD	surface-mounted device
SoC	system on chip
SOLT	short-open-line-through
SoP	system on package
TDR	time domain reflectometry
TEM	transverse electromagnetic
TET	Theoretische Elektrotechnik
TRL	through-reflect-line
TSV	through-silicon via
TUHH	Hamburg University of Technology
UQ	uncertainty quantification
via	vertical interconnect access
VNA	vector network analyzer



# Symbols, Notation, and Units

$A$	(typically) scalar
$(A)$	(typically) vector
$[A]$	(typically) matrix
$a_i$	incident power wave
$\hat{a}_i$	expansion coefficient
$B$	fractional bandwidth
$b_i$	reflected power wave
$C$	capacitance
$C'$	per-unit-length capacitance
$c_0$	speed of light
$d_{array}$	via pitch in arrays
$d_{gnd}$	distance to ground vias
$E$	electric field strength
$f$	frequency
$f_c$	cutoff frequency
$g_i$	normalized impedance
$H$	magnetic field strength
$H_{cav}$	cavity height
$H_{ms}$	microstrip height
$H_{sl}$	stripline height
$h$	scaling factor
$h_i$	normalized admittance
$I$	electric current
$I_d$	displacement current
$I_r$	return current
$\mathbf{i}$	multi-index
$j$	imaginary unit
$L$	inductance
$L'$	per-unit-length inductance
$l_m$	length of miter section
$l_{ms}$	microstrip length
$l_{qw}$	length of quarter-wave transformer

## Symbols, Notation, and Units

$l_{sl}$	stripline length
$l_{via}$	via length
$N$	filter order
$N_{cav}$	number of cavities
$N_{dd}$	number of differential ports
$N_{fences}$	number of via fences
$N_{gnd}$	number of ground vias
$N_{se}$	number of single-ended ports
$P$	order of approximation
$R$	pass-band ripple
$r_{apad}$	via antipad radius
$r_{gnd}$	ground via radius
$r_{pad}$	via pad radius
$r_{via}$	(signal) via radius
$S_{ij}$	scattering parameter
$\tan \delta$	dielectric loss tangent
$t_{pl}$	reference plane thickness
$v_{sl}$	stripline propagation velocity
$v_{via}$	via propagation velocity
$W_{eff}$	effective microstrip width
$W_{ms}$	microstrip width
$W_{qw}$	width of quarter-wave transformer
$W_{sl}$	stripline width
$Y$	admittance
$Z$	impedance
$Z_0$	reference port impedance
$Z_{ms}$	characteristic microstrip impedance
$Z_{qw}$	impedance of quarter-wave transformer
$Z_{sl}$	characteristic stripline impedance
$Z_{via}$	characteristic via impedance
$\beta_{via}$	via phase constant
$\gamma$	propagation constant
$\gamma_i$	univariate norm
$\gamma_{\mathbf{i}}$	multivariate norm
$\gamma_{via}$	via propagation constant
$\epsilon$	permittivity
$\epsilon_0$	permittivity of free space
$\epsilon_r$	relative permittivity
$\epsilon_{r,eff}$	effective relative permittivity of microstrips
$\eta_0$	impedance of free space

$\theta$	(auxiliary) angle
$\lambda$	wavelength
$\lambda_{via}$	guided wavelength of via
$\mu$	permeability
$\mu_0$	permeability of free space
$\mu_r$	relative permeability
$\xi$	random variable
$\boldsymbol{\xi}$	vector of random variables
$\xi_k$	Gaussian quadrature node
$\rho$	probability density function
$\boldsymbol{\rho}$	joint probability density function
$\sigma$	conductivity
$\phi$	polynomial basis function
$\boldsymbol{\phi}$	joint polynomial basis function
$\Omega_c$	normalized frequency
$\omega$	angular frequency
$\omega_k$	Gaussian quadrature weight

This work uses both the metric and imperial system of units. The conversion between the two unit systems corresponds to  $25.4 \mu\text{m} = 1 \text{ mil} = 0.001 \text{ in.}$