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Fatigue of the Tension-Stiffening Effect in Reinforced Concrete

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FATIGUE OF THE TENSION-STIFFENING EFFECT IN REINFORCED CONCRETE

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Foreword

The computational treatment of the bond between concrete and embedded reinforcing steel bars continues to be the subject of a debate among experts, even after many years of research. This is due to the complexity of the problem, as well as to the further development of construction methods and materials. In addition, with the focus on the assessment of existing structures, the expectations regarding the accuracy of predictions have also changed.

The dissertation presented by Tomás Arana Villafán deals with the bond action under fatigue loading and the influence of its degradation on the stiffness and the deformation capacity of tension chords and the shear resistance of girder webs. The investigations are based on a carefully conducted literature study as well as on tests on tensile elements under fatigue loading carried out at the TUHH. The Tension Chord Model and the method of Generalized Stress Fields serve as an uniform basis for theoretically addressing the issues.

The work is divided into six chapters: The introduction is followed by an overview on stochastically distributed actions and their computation for the example of (large) offshore structures. After that, a comprehensive discussion of the material properties of steel and concrete as well as of the bond behavior under static and repeated loading is presented. The main question of the thesis is pursued in the subsequent chapters. These start with the report on the tests with orthogonally reinforced tension members. Based on the results, the Tension Chord Model is extended for loading and unloading processes by adapting the corresponding design values and by introducing a linear damage function. Exemplarily, the findings are applied to the shear resistance of girder webs; the calculations are compared with experiments from the literature and good agreement is achieved. The work ends with a summary and conclusions.

This dissertation is an important contribution to the knowledge on the fatigue behavior of reinforced concrete structures. Tomás Arana Villafán critically reviews the assumptions made and the results found; in doing so, he points out deficiencies that still exist. Overall, he achieves results of high value for science and engineering practice. These represent a helpful basis for further research.

Lucerne (Switzerland), 2021
Prof. Dr. Viktor Sigrüst

Summary

The deformation and carrying capacity of concrete structures depends on the existing bond between concrete and reinforcement. Due to the action of fatigue loads, the bond experiences a degradation process which subsequently modifies the mechanical behaviour of reinforced concrete. In order to quantify the effects of a bond fatigue, diverse tests on concrete chords were performed and evaluated in the present work. A main focus lay on the application of realistic random loads, derived from various sea spectra, since the load history has proven to decisively influence the fatigue behaviour of concrete and steel. Therefore, fatigue tests should reflect the totality of expected loads on a structural element. Additionally, the biaxial stress state around reinforcing bars was systematically varied with the aim of measuring the impact of transverse tension on bond.

The tests results reveal a progressive deterioration of the bond. In consequence, the axial stiffness of tension chords is reduced while the value of crack width after loading increases and the remaining crack width after unloading decreases. A transverse tension does not affect the response behaviour of the tested specimens. Also the ultimate carrying capacity is not negatively altered. However, a higher structural deformation capacity, caused by a weakening of the tension-stiffening effect, could be observed.

In addition, a sort of plastic-strain-accumulation effect in reinforcing bars could be registered. Although the applied loads did not exceed the yield strength f_{sy} of steel, the reinforcement in most of the tested specimens showed continuously growing plastic deformations. After discussing possible causes, a linear equation is proposed for a quantification of this effect. Further research is required in order to either confirm or refute the development of plastic strains in reinforcement under fatigue loading.

In a further step the nonlinear development of bond degradation is linearised and the Tension Chord Model [115] for static-monotonic loading modified for a mathematical description of the fatigue process. In the proposed model, the rigid-plastic character of bond stresses is kept. The value of bond stresses at serviceability level is linearly reduced depending on the experienced number of load cycles. The reduction is done following the decrease ratio of the tension-stiffening.

Based on the developed model, also a modification of the Generalised Stress Field Analysis [177] is proposed in order to quantify the inclination Θ_{fat} of compression struts in concrete beams with web reinforcement under fatigue loading. Results of tests published in the scientific literature reveal a significantly flatter inclination of Θ_{fat} as stipulated in design standards, which base on linear stress field analysis. The proposed modification delivers more accurate values of Θ_{fat} and enables a more favourable design of beams elements under fatigue.

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Dresden, 2021
Tomás Arana Villafán

Notation

Roman capital letters

A_1	upper limit of the ferrite / cementite phase field
$A_{c,ef}$	effective concrete area
A_{ci}	idealised concrete area
A_{cn}	net concrete area
$A_{c,red}$	reduced cross sectional area
A_R	projected area of single rib
A_s	bar cross sectional area
$A_{s,fat}$	effective bar cross sectional area
A_{sz}	cross sectional area of reinforcement in z-direction
\underline{B}	damping matrix
C	constant
C_1	parameter
C_a	added mass coefficient
CC	consequence class
C_D	drag coefficient
C_{Ds}	drag coefficient for stationary flows
C_m	inertia coefficient
D	diameter, dimensionless damage
D_{equ}	real load-induced damage
D_{koll}	damage induced by $\Delta\sigma_{s,equ}$
EA	normal stiffness
E_{agg}	aggregate modulus of elasticity
E_c	concrete modulus of elasticity
$E_{c,fat}$	concrete modulus of elasticity under fatigue loading
E_{cm}	concrete secant modulus of elasticity
E_{c0m}	concrete tangent modulus of elasticity
E_i	idealised modulus of elasticity of uncracked concrete chord
E_{sm}	effective modulus of elasticity of tension chord
E_{sm0}	effective modulus of elasticity of tension chord at load beginning
F	force
\underline{F}	force matrix
F_{cr}	crack-inducing force
F_{hyd}	hydrodynamic force
F_{ins}	instationary force

F_{xV}	force component of stress field in x-direction
G	general failure function
G_f	dissipated energy per unit area
H	wave height, transfer function
HCF	high cycle fatigue
H_s	significant wave height
K	wave number, stress concentration range
\underline{K}	stiffness matrix
KC	Keulegan-Carpenter number
LCM	low cycle fatigue
M	bending moment
\underline{M}	mass matrix
N	normal force, number of loads
N_0	normal force range
N_f	total number of load cycles until failure
N_u	number of loads cycles that leads to fatigue failure
N^*	number of load cycles where inclination of Wöhler curve changes
P_m	post-tensioning force
Q_k	characteristic value of variable load action
$Q_{k,0.98}$	98% quantile of characteristic value of variable load action
QTF	quadratic transfer function
R	range, normal density function of resistance, response spectrum
RAO	response amplification factor
Re	Reynolds number
RC	reliability class
S	normal density function of load action, wave spectrum
$S_{c,a}$	relative amplitude compression strength
$S_{c,m}$	relative average compression strength
$S_{c,max}$	relative maximum compression strength
$S_{c,min}$	relative minimum compression strength
S_S	response spectrum
S_1	sea spectrum
T	period
T_c	mean wave period
T_{cyc}	load period
T_{mg}	melting temperature of steel
T_t	transition temperature which leads to creep in steel
T_p	peak wave period
T_z	zero-up crossing period of wave
$T_{z,oe}$	zero-up crossing period of response
U_{cF}	specific fracture energy
V	shear force
V_{fat}	shear force under fatigue loading
$V_{R,c}$	shear resistance capacity of web concrete

$V_{R,sy}$	shear resistance capacity of web reinforcement
Y	yield function
Y_c	yield function of plain concrete
Y_{c1}	first yield limit of plain concrete
Y_{c2}	second yield limit of plain concrete
Y_s	response transfer function
Roman lower case letters	
a	water acceleration, crack length
a_0, a_1, \dots, a_n	Fourier coefficients, parameters
\hat{a}	amplitude
a_{sx}	lengthwise cross sectional area of reinforcement in x-direction
a_{sw}	lengthwise cross sectional area of web reinforcement
a_{sz}	lengthwise cross sectional area of reinforcement in z-direction
b	damping coefficient
b_0, b_1, \dots, b_n	Fourier coefficients
b_{ffi_s}	parameter
b_\emptyset	parameter
b_w	web width
c_0, c_1	constant values
c_{nom}	concrete cover
c_s	internal concrete cohesion
d	static height
d_w	water depth
f	frequency
f_c	uniaxial concrete compression strength
$f_{c,cube}$	uniaxial concrete compression strength tested on a cube
f_{ce}	concrete effective compression strength
$f_{c,fat}$	uniaxial fatigue strength of concrete
f_{ct}	concrete uniaxial tension strength
$f_{ct;0.05}$	5%-quantile of concrete uniaxial tension strength
$f_{ct;0.95}$	95%-quantile of concrete uniaxial tension strength
f_{load}	load frequency
f_R	bond index
f_{su}	steel uniaxial ultimate strength
f_{sy}	steel uniaxial yield strength
f_t	uniaxial tension strength
g	gravity constant
k	coefficient of f_{py}/f_{sy} , inclination of Wöhler curve
k_t	reduction factor
k_\emptyset	factor for quantification of τ_{bU} and τ_{bR} in dependence of f_{ct}
k_u	displacement coefficient, coefficient of steel stress at cracked section by reloading and at first cracking

l_b	bond length
l_{by}	transmission length by yielding of reinforcement
m	exponent for Wöhler curve
m_a	added mass
m_0, m_2, m_4	statistical moments
m_{pl}	slope of $\varepsilon_{sm,pl} - n/N$ -curve
m_{sm}	slope of $E_{sm} - n/N$ -curve
m_{yy}	lengthwise bending moment
n	natural number, coefficient of E_s/E_c
n_{equ}	equivalent number of load cycles
n_x	axial membrane forces in x-direction
n_z	axial membrane forces in z-direction
\varnothing_s	reinforcing bar diameter
p	pressure, probability
p_0	atmospheric pressure
p_f	failure probability
p_{ins}	instationary pressure
p_r	radial compression
r_1, r_2	parameters of meridians
r_c	radius function
r_i	radius of inner concrete ring
r_o	radius of outer concrete ring
s_{rm}	average crack spacing
$s_{rm,max}$	maximum average crack spacing
$s_{rm,min}$	minimum average crack spacing
s_{r0}	maximum distance between cracks
u	mode value, velocity in x-direction
u_1	mode value in a reference period of 1 year
u_{50}	mode value in a reference period of 50 years
u_a	amplitude velocity of oscillating fluid
v	velocity in y-direction
w	velocity in z-direction
w_{cr}	crack width
$w_{cr,lim}$	limit value of crack width
w_{tr}	transverse crack width
w_u	crack width by which no force transmission is possible
x_{sR}	intersection point of steel stress distribution between unloading and reloading process
x_{sU}	intersection point of steel stress distribution between loading and unloading process
\underline{x}	deformation or motion matrix
$\underline{\dot{x}}$	velocity matrix
$\underline{\ddot{x}}$	acceleration matrix

\dot{x}_r	velocity matrix of rigid body
\ddot{x}_r	acceleration matrix of rigid body
z	lever arm of internal forces
Greek letters	
$\alpha_1, \alpha_2, \alpha_3$	parameters
α_b	inclination angle of stress resultant around reinforcing bar
$\alpha_{c,duc}$	ductility degree factor
α_e	$= E_s/E_c$
α_i	parameter
β_b	inclination angle between reinforcing bar and concrete wedge
$\beta_{c,fat}$	$= \varepsilon_{c,fat,da}/\varepsilon_{c,fat}$
β_{RC}	reliability index
β_i	mean direction of sea state
γ	shearing strain
γ_Q	safety factor for variable load action
$\gamma_{s,fat}$	material safety factor at fatigue limit state
γ_{Sd}	load safety factor at fatigue limit state
γ_{xz}	shearing strain in xz-plane
$\Delta\varepsilon_{s,el}$	elastic steel strain range
$\Delta\varepsilon_{s,pl}$	plastic steel strain range
$\Delta\varepsilon_{sm0}$	difference of average steel strain resulting from degradation of tension-stiffening
$\Delta\sigma_s$	normal stress range
$\Delta\sigma_{s0}$	$= N_0/A_s$
$\Delta\sigma_{s,equ}$	damage equivalent stress range
δ_d	factor for steel ductility
δ_s	slip between reinforcing bar and surrounding concrete
δ_{s0}	slip δ_s under static-monotonic loading
δ_{s1}	slip δ_s after one load cycle
δ_{sr}	residual slip
$\delta_{s,max}$	maximum bond slip
ε_1	principal strain
ε_3	principal strain
ε_c	concrete strain
$\varepsilon_{c1}, \varepsilon_{c3}$	concrete principal strains
ε_{c0}	uniaxial concrete compression strength at failure
$\varepsilon_{c3,el}$	elastic component of concrete strain under fatigue
$\varepsilon_{c3,t}$	time-dependent component of concrete strain under fatigue
$\varepsilon_{c,fat}$	concrete strain under fatigue loading
$\varepsilon_{c,fat,da}$	damage-induced strain under fatigue
ε_{cm}	average concrete strain
ε_{ct}	$= f_{ct}/E_c$

ϵ_{cu}	concrete failure strain
$\epsilon_{cu,fat}$	concrete ultimate fatigue strain
ϵ_n	phase angle
ϵ_r	remaining strain after unloading
ϵ_{sm}	average steel strain
$\epsilon_{sm,cal}$	calculated average steel strain
$\epsilon_{sm,mes}$	measured average steel strain
$\epsilon_{sm,\theta}$	bond-dependent average steel strain
$\epsilon_{sm,pl}$	plastic part of average steel strain
$\epsilon_{s,max}$	maximal steel strain
ϵ_{smz}	average steel strain in z-direction
ϵ_{sr}	steel strain at at midpoint between cracks
ϵ_{sr0}	steel strain at cracked section
ϵ_{srx}	steel strain at cracked section in x-direction
ϵ_{srz}	steel strain at cracked section in z-direction
ϵ_{sy}	uniaxial steel yield strain
ϵ_{uf}	strain of flange of a composite bridge
ϵ_x	strain in x-direction
ϵ_z	strain in z-direction
ζ	water level function
ζ_a	wave amplitude, response amplitude
η_D	limit of cumulative damage ratio
η_δ	parameter for quantification of slip reversal
η_{TS}	tension-stiffening number
Θ	inclination angle
θ	relative roughness
Θ_c	Haigh-Westergaard-coordinate
Θ_{fat}	inclination of compression strut at fatigue limit state
$\Theta_{fat,cal}$	calculated inclination of compression strut at fatigue limit state
$\Theta_{fat,MC2010}$	inclination of compression strut at fatigue limit state according to Model Code 2010
$\Theta_{fat,mes}$	measured inclination of compression strut at fatigue limit state
$\Theta_{FB,stat}$	inclination of compression strut at ultimate limit state according to DIN Fachbericht 102
Θ_{max}	maximal inclination angle
Θ_{min}	minimal inclination angle
Θ_{ult}	inclination of compression strut at the ultimate limit state
Θ_z	yaw motion
K_{b0}	bond strength according to Tension Chord Model
$K_{c,fat}$	damage parameter
K_{rsd}	calculation factor of residual bond strength
$K_{t,fat}$	damage parameter
K_τ	bond coefficient

λ	parameter for quantification of distance between cracks
λ_w	wave length
μ	mean value
μ_c	elastic Poisson's ratio
μ_G	mean value of G
μ_{H_s, σ_s}	median value of significant stress in reinforcement
μ_R	mean value of R
μ_S	mean value of S
ν	Poisson's number
ξ	parameter
ξ_c	Haigh-Westergaard-coordinate
ρ_c	Haigh-Westergaard-coordinate
$\rho_{s,ef}$	effective geometrical reinforcement ratio
ρ_{sx}	geometrical reinforcement ratio in x-direction
ρ_{sz}	geometrical reinforcement ratio in z-direction
ρ_w	water density
σ	normal stress
σ_1	principal stress
σ_3	principal stress
σ_{br}	radial stress
$\sigma_{b\phi}$	circumferential stress
σ_c	concrete normal stress
σ_{c1}	concrete principal stress
σ_{c3}	concrete principal stress
$\sigma_{c,D}$	real stress due to fatigue-induced damage of cross section
σ_{cx}	concrete normal stress in x-direction
σ_{cz}	concrete normal stress in z-direction
$\bar{\sigma}_s$	mean steel stress
σ_{H_s, σ_s}	standard deviation of significant stress in reinforcement
$\sigma_{s,max}$	maximal steel stress
σ_{sr}	steel stress at cracked section
σ_{sr0}	steel stress at cracked section immediately after crack formation
σ_{std}	standard deviation
σ_{sx}	steel stress in x-direction
σ_{sw}	steel stress in web reinforcement
σ_{sz}	steel stress in z-direction
σ_x	normal stress in x-direction
σ_z	normal stress in z-direction
τ	shear stress
τ_b	bond strength
τ_{bf}	frictional bond strength
$\tau_{b,0.1}$	bond stress at a slip of 0.1 mm
τ_{b0}	rigid-plastic bond strength for $\sigma_s < f_{sy}$
τ_{b1}	rigid-plastic bond strength for $\sigma_s \geq f_{sy}$

$\tau_{b,max}$	ultimate bond strength
τ_{bR}	rigid-plastic bond stress by reloading
τ_{bR1}	rigid-plastic bond stress by 1 st cycle of reloading
$\tau_{bR,rsd}$	residual rigid-plastic bond stress by reloading
τ_{bU}	rigid-plastic bond stress by unloading
τ_{bU1}	rigid-plastic bond stress by 1 st cycle of unloading
$\tau_{bU,rsd}$	residual rigid-plastic bond stress by unloading
τ_c	concrete shear stress
τ_{cxz}	concrete shear stress in the xz-plane
$\tau_{R,max}$	ultimate shear stress
τ_{xz}	shear stress in the xz-plane
Φ	potential function
Φ_b	conical shell expansion of bond stresses
ϕ_c	angle of internal friction of concrete
Φ_n	phase angle
Φ_s	potential function of diffracted wave
Φ_w	potential function of undisturbed wave
Φ_x	roll motion
Ψ_{KC}	modification factor for instationary flows
Ψ_y	pitch motion
ψ_0, ψ_1, ψ_2	combination factor
ω	circular frequency
ω_{sy}	mechanical reinforcement ratio
ω_p	circular peak frequency
ω_{py}	mechanical ratio of prestressing steel
ω_T	wave circular frequency