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**Reza Hashemi**

**Optimizing Control and Process  
Monitoring of a Continuous Poly-  
merization Process in Tubular  
Reactors**

# **Optimizing Control and Process Monitoring of a Continuous Polymerization Process in Tubular Reactors**

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Ludwigshafen am Rhein, 27.09.2019

*Reza Hashemi*



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# Abstract

With the emergence of high performance computers and the constant increase of their computation power and the availability of efficient numerical optimization algorithms, model-based optimizing control schemes receive great attention. The conceptual simplicity of these schemes and their capability to deal with constraints and handle nonlinear systems as well as the systems with multiple inputs and outputs make this method attractive in theory and practice.

The focus of this thesis is to study the different aspects of the application of optimizing controller on a continuous polymerization process of acrylic acid in tubular reactors. This process is described by a system of PDEs comprising 8 – 12 manipulated variables and is characterized by complex behavior, long delays and sharp evolution of the states. Several numerical methods are used to simulate this process to figure out the approach with the highest accuracy and the lowest computation times.

To promote a real-time implementation, a computationally favorable formulation of an optimizing controller for the continuous polymerization process of acrylic acid is proposed and its different aspects are studied in several control scenarios.

The availability of state information and their accuracy are critical for the applicability of optimizing controllers. It is difficult to estimate the states from the available measurements for distributed systems such as the continuous polymerization process. Therefore different estimation methods are studied and compared in terms of estimation accuracy, tuning effort and computation times to determine the most suitable method for this process.

The quality of the model used by the optimizing controller influences its performance substantially. To achieve robustness against the model inaccuracy, the standard Sequential Importance Resampling Particle Filter is adapted such that simultaneous estimation of the states and the parameters of the continuous polymerization process is possible. The proposed method can also be used for any process with long input-output delays.





# Zusammenfassung

Durch die Entwicklung von Hochleistungscomputern und durch den konstanten Anstieg ihrer Rechenleistung sowie durch die Verfügbarkeit effizienter numerischer Optimierungsalgorithmen wurde der modellbasierten ökonomisch-optimierenden Regelung viel Aufmerksamkeit geschenkt. Die konzeptionelle Einfachheit dieser Methode und ihre Vorteile durch die Handhabung nichtlinearer Systeme mit mehreren Stell- und Regelgrößen sowie die Realisierung von Beschränkungen, macht diese Methode aus theoretischen und praktischen Gesichtspunkten interessant.

Der Fokus der vorliegenden Dissertation wurde auf die Betrachtung verschiedener Aspekte der Anwendung von ökonomisch optimierender Regelung an einem kontinuierlichen Polymerisationsprozess zur Herstellung von Polyacrylsäure gelegt. Dieser Prozess wird in einem Rohrreaktor durchgeführt und kann mit Hilfe von 8-12 Stellgrößen geregelt werden. Das Systemverhalten wird mit Hilfe von PDEs abgebildet und besitzt stark nichtlineares dynamisches Verhalten, bei dem lange Verzugszeiten und scharfe Fronten sowie Konzentration-Peaks auftreten. Verschiedene Alternativen werden für die numerische Simulation angewendet um die Methode, die am besten geeignet ist, zu identifizieren.

Um eine Echtzeit-Implementierung zu fördern, wird eine vorteilhafte mathematische Beschreibung des optimierenden Reglers für die kontinuierlichen Herstellung von Polyacrylsäure vorgeschlagen und ihre verschiedenen Aspekte werden durch verschiedene Regelungsszenarien studiert.

Die Verfügbarkeit der Information über Systemzustände sowie deren Genauigkeit sind kritisch für die Anwendung der optimierenden Regelung. Der Zustandsschätzung aus den vorhanden Messungen ist in verteilten Systemen wie zum Beispiel die kontinuierliche Herstellung von Polyacrylsäure schwierig. Daher werden Methoden zur Zustandsschätzung im Rahmen der Arbeit untersucht und hinsichtlich der Schätzgenauigkeit, des Einstellaufwands und der Berechnungszeiten verglichen, um die bestmögliche Methode auszuwählen und für den vorliegenden Prozess zu nutzen.

Die Qualität des Prozessmodells, das für den optimierenden Regler genutzt wird, hat einen wichtigen Einfluss auf Performance des Reglers. Um robust gegenüber Modellunsicherheit zu sein, wurde der Standard Sequential Importance Resampling Partikelfilter als Methode zur Zustandsschätzung so angepasst, dass gleichzeitig Systemzustände und -parameter geschätzt werden können. Somit kann der verwendete Zustandsschätzer auch auf andere Prozesse, die durch lange Verzögerungszeiten zwischen Stell- und Regelgrößen charakterisiert sind, übertragen werden.



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