

Berichte aus der Physik

**Peter Machon**

**Electronic Transport Theory  
of the Spin-Dependent Proximity Effect  
in Superconductor-Ferromagnet Heterostructures**

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## Preface

This thesis is a selection of my most notable works, performed during my time at the Universität Konstanz, in the Quantum Transport Group, supervised by Prof. Wolfgang Belzig. The thesis has a short introduction, outlining the most important formulas and laws that are necessary to follow the main work. This introduction is not intended to be a complete derivation of all theories used in the work, but rather should give an overview and provide references to more detailed sources. The main work, starting from chapter 2, consists of papers that are published (chapters 4<sup>1</sup> and 5<sup>2</sup>), submitted <sup>3</sup> (chapter 2) and are under development (chapters 3 and 6). Thus, each chapter contains a separate abstract and introduction to the special topics discussed in it, and notations are not necessarily consistent among chapters. The articles are not included in chronological but in a logical order, chosen by the author, and are not identical with the published version. This ordering scheme was used, mainly to underline connections between chapters.

In addition to Prof. Belzig provided supervision and strong contribution to this research, it is important to recognize, that parts of this thesis were developed in collaboration with Prof. Matthias Eschrig, Prof. Elke Scheer and PD Detlef Beckmann (and their respective group members). The latter two contributors are mentioned at the corresponding places. M. Eschrig was a key collaborator and second author of the papers that are the basis for chapter 4 and 5 and was in addition part of discussions about the work in chapter 2. He thus has a highly valuable and quite extended contribution to

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<sup>1</sup>P. Machon, M. Eschrig, and W. Belzig, *Phys. Rev. Lett.* **110**, 047002 (2013).

<sup>2</sup>P. Machon, M. Eschrig, and W. Belzig, *New Journal of Physics* **16** (2014) 073002.

<sup>3</sup>P. Machon and W. Belzig, arXiv:1502.05567 (2015).

the present thesis. Since it is not possible to separate in detail which word was written by whom without altering the internal logic of this work, I will instead outline the biggest contributions of M. Eschrig in this thesis that were not produced by myself. These points and the associated sections of this thesis are listed below:

- The derivation of the extended dirty limit tunnel boundary condition (after first together realizing and correcting the issues in [1], what can in an equivalent formulation be found in the corresponding erratum).
- The derivation of the clean limit current contributions and the core program to calculate them.
- Section 4.7.1 that describes in detail the processes in the clean limit.
- The pictures in section 4.7.5 that compared all different formulations of the nonlocal thermopower in the clean limit, all in chapter 4.
- The analytical proof of Onsagers symmetry in the clean limit (section 4.7.2) were calculated together, first written by me, and finished together.
- In addition M. Eschrig and W. Belzig had the basic ideas for chapter 4 and both strongly contributed to the development of the proposed experiment in chapter 5, while W. Belzig of course had this strong contribution in all parts of this thesis.

To conclude this preface, I would like to mention that this thesis contains both, analytical and numerical parts. All analytical calculations were done exclusively for this thesis. Also all numerical programs that were used in this thesis, have been developed from scratch solely by the author and the aforementioned contributor. The developed numerical simulations are dependent upon the following open source libraries: lapack, blas, quadpack<sup>4</sup> and mpi, and on a root finding algorithm based on Brent's method, that was found on the netlib ([www.netlib.org](http://www.netlib.org)).

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<sup>4</sup>... that needed a bugfixing done by myself.

## Statutory Declaration

I declare that I have developed and written the enclosed dissertation completely by myself, and have not used sources or means without explicit declaration. Any thoughts from others or literal quotations are clearly marked or stated above. The dissertation was not used in the same or in a similar version to achieve an academic grading or is being published elsewhere.

Konstanz,

Date, Peter Machon



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## Acknowledgments

This work is dedicated to my grandfather, who sadly passed away during my stay at the Universität Konstanz. I want to thank him, as well as my mom and grandmother for always unconditionally supporting me.

Special thanks goes to my supervisor and boss Prof. Wolfgang Belzig for the invaluable amount of trust, patience and guidance throughout the whole last five and a half years, and for always being available for useful discussions. I would like to mention, that at the beginning of my PhD time, I had no clue about the interesting field of quantum transport at all. In addition, after almost two years without a job in science I was about to get lost on my way, but Wolfgang gave me the great opportunity to proof myself and come back into science.

I would like to thank Prof. Matthias Eschrig for contributing this huge amount of work, his great ideas, and being up for discussions at literally each day and night time.

Also, I would like to thank all the people that made my stay here being such an enjoyable time: My two long-time flatmates Jens Sikeler and Carolin Schuster, and all my colleagues and especially also new friends in- and outside the university (to many to list). Especially, next to my group, I want to thank the Burkard group for treating me like a full member of their group, and apparently always waiting for me with a fresh piece of cake and a super funny new wordplay.





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# Contents

<b>Preface</b>	<b>i</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation und Kurzfassung . . . . .	1
1.2 Motivation and Abstract . . . . .	3
1.3 Green's functions in solid state physics . . . . .	5
1.3.1 The many-body propagator . . . . .	5
1.3.2 Fermi liquid theory . . . . .	6
1.4 Quasiclassical theory . . . . .	7
1.4.1 BCS Theory in the Gorkov Technique . . . . .	7
1.4.2 The quasiclassical approximation, i.e. the Eilenberger Equation . . . . .	7
1.4.3 The dirty limit, i.e. the Usadel Equation . . . . .	9
1.5 Keldysh technique . . . . .	10
1.6 Superconducting Heterostructures . . . . .	13
1.6.1 The proximity Effect . . . . .	13
1.6.2 Superconductivity vs. Ferromagnetism . . . . .	14
1.6.3 Spin-dependent scattering Theory . . . . .	15
1.6.4 Boundary conditions . . . . .	17
1.6.5 Quantum circuit theory . . . . .	20

<b>2</b>	<b>Fully spin-dependent BC for isotropic quasiclassical Green's functions</b>	<b>23</b>
2.1	Abstract . . . . .	23
2.2	Introduction . . . . .	23
2.3	Spin separation of the transfer matrix . . . . .	25
2.4	Generalized spin-dependent boundary condition . . . . .	27
2.5	Strongly spin-dependent tunnel contacts . . . . .	30
2.6	Conclusion . . . . .	31
<b>3</b>	<b>Evidence for strong spin mixing at FI-S interfaces</b>	<b>33</b>
3.1	Abstract . . . . .	33
3.2	Introduction: Ferromagnetic Insulator - Superconductor heterostructures . . . . .	33
3.3	The experiment represented by the circuit theory . . . . .	35
3.4	Circuit theory for FI-S bilayers with exact spin-dependent boundary conditions . . . . .	36
3.5	General properties of FI-S structures . . . . .	37
3.6	Comparison: Theory vs. Experiment . . . . .	42
3.7	Conclusions . . . . .	45
<b>4</b>	<b>Nonlocal thermoelectric effects in a three terminal S-F device</b>	<b>47</b>
4.1	Abstract . . . . .	47
4.2	Introduction . . . . .	47
4.3	Linear response description of three terminal devices . . . . .	49
4.4	The formalism . . . . .	50
4.4.1	Scattering approach for spin-active Interfaces . . . . .	50
4.4.2	Quantum circuit theory, i.e. The dirty limit . . . . .	51
4.4.3	Ballistic trajectories, i.e. The clean limit . . . . .	51
4.5	The Thermopower (local and nonlocal) . . . . .	53
4.5.1	The basic mechanism . . . . .	53
4.5.2	The nonlocal Thermopower, clean vs. dirty limit . . . . .	54
4.5.3	Generalization of Onsager's reciprocal relation to non-local currents . . . . .	57
4.6	Conclusion . . . . .	57
4.7	Detailed proofs and further discussions . . . . .	57
4.7.1	Classification of local and nonlocal processes for ballistic three-terminal systems . . . . .	57
4.7.2	Analytic proof of Onsager's symmetry in the clean limit . . . . .	58
4.7.3	Vanishing of pure Andreev reflection contributions to the thermoelectric effects in the clean limit . . . . .	61

4.7.4	Nonlocal origin of all thermoelectric coefficients in the clean limit . . . . .	62
4.7.5	Various definitions of nonlocal thermopower . . . . .	63
<b>5</b>	<b>Giant thermoelectric effects in a proximity-coupled S-F device</b>	<b>67</b>
5.1	Abstract . . . . .	67
5.2	Introduction . . . . .	67
5.3	The Model . . . . .	70
5.4	Spin-dependent quasiclassical theory in the dirty limit . . . . .	72
5.5	Thermopower . . . . .	75
5.6	Figure of Merit . . . . .	80
5.7	Temperature Dependence . . . . .	83
5.8	Conclusions . . . . .	85
<b>6</b>	<b>Numerical implementation of the spin-dependent quantum circuit theory for arbitrary circuits in the tunneling limit and simulation of a real experiment</b>	<b>87</b>
6.1	Abstract . . . . .	87
6.2	Introduction . . . . .	88
6.3	Iterative solution of the circuit theory . . . . .	89
6.4	The numerical implementation . . . . .	94
6.4.1	Physically motivated optimizations . . . . .	94
6.4.2	The work principle and the user interface . . . . .	95
6.4.3	Numerical optimizations . . . . .	95
6.5	Simulation of the experiment . . . . .	97
6.5.1	The experimental setup . . . . .	97
6.5.2	The setup in the circuit theory . . . . .	98
6.5.3	Measured results . . . . .	99
6.5.4	Simulation and comparison . . . . .	101
6.6	Conclusion . . . . .	103
<b>7</b>	<b>Summary and Outlook</b>	<b>105</b>
<b>A</b>	<b>Derivation of the Wigner representation</b>	<b>109</b>
<b>B</b>	<b>Analytically solvable extension to chapter 3</b>	<b>113</b>
<b>C</b>	<b>Spin-caloric effects in a transistor like proximity structure</b>	<b>115</b>