

Analytic Source and Volume Conductor Models for Biomagnetic Fields

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Table of symbols

ε	dielectric constant
ε_0	dielectric constant of free space
μ	permeability
μ_0	permeability of free space
$\mathbf{E}$	electric field strength
$\mathbf{B}$	magnetic flux density
ϕ	sensitivity profiles
Φ	magnetic flux
V	electric potential
U	magnetic scalar potential
$\mathbf{A}$	magnetic vector potential
σ	conductivity
ρ	charge density
$\mathbf{j}$	current density
$\mathbf{j}^p$	primary current density
$\mathbf{d}$	current dipole
$\mathbf{a}_{10}, \mathbf{a}_{11}, \mathbf{b}_{11}$	current quadrupole coefficient vectors
$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$	current quadrupole coefficient vectors
S	surface
v	volume
$d\mathbf{s}$	infinitesimal surface element
d^3r	infinitesimal volume element
$\mathbf{r}$	location of field point
$\mathbf{r}_0$	location of source point
δ_{ij}	Kronecker delta function
δ	Dirac delta function
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$	Cartesian unit vectors
$\mathbf{e}_\eta, \mathbf{e}_\xi, \mathbf{e}_\phi$	prolate unit vectors
$\mathbf{e}_r$	radial unit vector
$\mathbf{n}$	normal unit vector

$\hat{\mathbf{I}}$	identity matrix
i, j, k, l, m, n	summation indices
x, y, z	Cartesian coordinates
r, Θ, φ	spherical coordinates
η, ξ, φ	prolate coordinates
f	focal length of prolate coordinates
a, b	long half axis and short half axis of a prolate spheroid
η_v	boundary of a prolate spheroid volume conductor
h_η, h_ξ, h_φ	metric coefficients of prolate coordinates
η_0, ξ_0, φ_0	source point in prolate coordinates
$R_{nm}, \Theta_{nm}, \Phi_{nm}$	functions of prolate coordinates η, θ, φ
ν, μ	degree and order of Legendre functions
n, m	integral degree and order of Legendre polynomials
P_{nm}, Q_{nm}	Legendre polynomials of 1st and 2nd kind of degree n and order m
$f_{nm}, \tilde{f}_{nm}$	expansion functions of Green's function
X_{nm}	spherical harmonics of degree n and order m
$\mathbf{a}_{nm}, \mathbf{b}_{nm}$	current multipole coefficient vectors
$O(x^n)$	function of order n in x
s	number of detectors
Q	source space
t	number of expansion coefficients
A	expansion coefficients
$\hat{\mathbf{P}}$	MFT system matrix
$\zeta, \tilde{\zeta}$	regularization parameter
w	probability weight
λ_l, λ_d	lateral and depth decay lengths of Gaussian probability weight