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Easy Linear Optimization

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Introduction

Linear Optimization deals with certain simple kinds of optimization problems: namely, the ones which can be formulated as finding the maximum or the minimum of some linear function of finitely many variables on a set defined by some linear inequalities over these variables. Such programs arise naturally in economics (e.g., minimize the expenditures such that the necessary production volume is still reached) and computer science (e.g., the question whether there is a valuation of variables satisfying a system of linear inequalities lies at the heart of many automatic theorem provers).

The area of linear optimization is well-studied since the works of L. Kantorovich[1] and G. Dantzig[2]. Most books on this area (in particular, the ones on the undergraduate level) fix the field of reals or rationals right from the start. While convenient, there is no need for such a restriction of the field. It turns out that it does not hurt to work over any subfield of the reals: the overwhelming majority of the theory and algorithms require no substantial change of the high-level view; one only has to pay attention to the low-level proofs.

The possibility to choose the field arbitrarily is practically important. To correctly implement computer algebra systems, we need to know which high-level algorithms are correct regardless of the field over which the low-level operations are implemented. Moreover, sometimes our implementations of the algorithms need to be precise. For example, if we wish to determine the dimension of a particular polyhedron, we need to work with an exact representation of the numbers rather than with their approximate floating-point representation. To work with such exact representations efficiently we need to know that the computations never leave the field from which the original data is taken. For example, if all the coefficients of a linear function and a system of linear inequalities lie in $\mathbb{Q}(\sqrt{2})$, then the maximum of the linear function over the (for simplicity, bounded) area determined by the linear inequalities also lies in $\mathbb{Q}(\sqrt{2})$, and the computations for determining this maximum never leave $\mathbb{Q}(\sqrt{2})$.

Our treatise takes such an approach: it develops the theory over an arbitrary subfield of the real numbers right from the start. This approach is an advantage for an undergraduate student, who typically already knows what a field and a subfield are from the initial lectures on linear algebra.

Related work

Presentation of the material over an arbitrary field also occurs in [10], which is, however, in German. Concerning the choice of material and its order, we partially use the ideas from [12], which is also in German. Opposed to that work, we discuss a smaller number of topics and present them in a more thorough, Bourbaki-like way, more targeted towards undergraduate students. Also, we introduce certain topics not present in the aforementioned book, for example,

[1] Леонид Витальевич Канторович, * January 6 (Julian calendar) / 19 (Gregorian calendar), 1912, St. Petersburg, Russian Empire – ‡ April 7, 1986, Moscow, USSR. A Soviet mathematician and economist, Nobel prize winner.

[2] George Bernard Dantzig, * November 8, 1914 – ‡ May 13, 2005. An American mathematical scientist, the developer of the Simplex algorithm.

a chapter containing the necessary mathematical background or the fully spelt-out computation of Klee-Minty cubes.

A similar style of introducing linear programming can be found in [10].

Proofs of certain claims are self-constructed, while ideas for certain others come from [12], [10], and [16].

Disclaimer

Our work is not intended to be self-contained: certain material that is present in other literature in sufficient quality as well as certain standard parts of set theory and linear algebra are not necessarily included.