





# **Nonlinear State and Parameter Estimation for Small Unmanned Aircraft Systems**

A thesis accepted by the Faculty of Aerospace Engineering and Geodesy of the  
University of Stuttgart in partial fulfillment of the requirements for the degree of  
Doctor of Engineering Sciences (Dr.-Ing.)

by

**Florian Weimer**

born in Stuttgart, Germany

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|------------------|---|
| Main Referee:    | Prof. Dr.-Ing. Walter Fichter                                   |
| Co-Referee:      | Prof. Eric N. Johnson Ph.D.,<br>Georgia Institute of Technology |
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**Florian Weimer**

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Internet: [www.shaker.de](http://www.shaker.de) • e-mail: [info@shaker.de](mailto:info@shaker.de)

*To Lea!*





# Preface

When I started working at the Institute of Flight Mechanics and Control, there was nothing but the basic idea to establish an sUAS research group with a strong focus on real world demonstrations. As we started from scratch, I spend much time in building up an adequate development environment and learned a lot on digital circuits, PCB design, and soldering. Although this was quite enjoying, it did not bring me any closer to serious research contributions. At some point in time, I even was not sure if I would ever submit a thesis, and now it is hard for me to believe that I managed to get that far.

First of all, I would like to thank my supervisor Walter Fichter. All that would not have been possible without his excellent guidance and support. Besides the fruitful technical discussions he always encouraged me to carry on. Moreover, many thanks to Eric N. Johnson for taking the responsibility of acting as co-referee of this work. It was a real honor that he made all the way from Atlanta, Georgia to Stuttgart to join my defense.

Furthermore, I would like to thank the staff at the Institute of Flight Mechanics and Control for the wonderful time I had. It was a really pleasant working atmosphere and I enjoyed it a lot! My special thanks go to André Posch and Michael Frangenberg for their great commitment in the hardware and software development of the onboard computing system. Both of them contributed significantly to the numerous successful flight tests both in the scope of this thesis and for several other projects.

Last but not least I want to thank my wife Gisela and my daughter Lea for their endless patience and encouragement. You two are the most important thing in my life.

Stuttgart, February 2016  
*Florian Weimer*



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