

Multi-scale Model for Fatigue in Carbide-rich Tool Steel

Von der Fakultät für Maschinenwesen der Rheinisch-Westfälischen Technischen Hochschule Aachen zur Erlangung des akademischen Grades eines Doktors der Ingenieurwissenschaften genehmigte Dissertation

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ZUSAMMENFASSUNG

Karbideiche Werkzeugstähle werden nicht nur in der Werkzeugindustrie, sondern auch für Maschinenelemente wie z.B. Lager, Dieselinjektoren und Verbindungselemente eingesetzt. Bauteile aus diesen Materialien sind häufig zyklischer Beanspruchung ausgesetzt. Ermüdung ist als wichtigste Versagensursache anzusehen, weil das Versagen ungefähr 90% aller metallischen Bauteile auf diese Fehlerursache zurückzuführen ist [1]. Werkzeugstähle sind ebenfalls empfindlich gegenüber schwingender Beanspruchung. Die Ermüdfestigkeit dieser Werkstoffe hängt zudem empfindlich von den mikrostrukturellen Eigenschaften wie der Form, dem Größenverhältnis, dem Volumenanteil und der Verteilung von primären und eutektischen Karbiden ab. Deshalb, werden neben dem Lastkollektiv die mikrostrukturellen Eigenschaften als die wichtigsten Einflussfaktoren auf die Lebensdauer von Werkzeugkomponenten angesehen.

Es ist bekannt, dass die Lebensdauervorhersage von Karbidreichen Werkzeugstählen unter schwingender Beanspruchung keine einfache Aufgabe ist. Aus diesem Grund ist es wichtig eine Wissensbasis über die Einflüsse der mikrostrukturellen Eigenschaften der Werkstoffe auf das Versagensverhalten aufzubauen. Das Hauptziel dieser Arbeit besteht darin schrittweise ein einfaches Modell sowie einen Rahmen zur numerischen Untersuchung der Einflüsse der mikrostrukturellen Eigenschaften auf die Langzeitfestigkeit (HCF) zu erstellen.

Im Allgemeinen kann die Ausbreitung des Schwingbruchs in drei Stufen eingeteilt werden: Die anfängliche Rissbildung (Rissinkubation), Kurz- und Langrisswachstum. Diese drei Stufen des Risswachstums wurden von McDowell [2] erfolgreich in einem so genannten mehrskaligen Versagensmodell (MSF) abgebildet. Um das Ermüdungsverhalten von Karbidreichen Werkzeugstählen zu modellieren, wurde das Modell von McDowell modifiziert und weiterentwickelt, um drei Längenskalen in Betracht ziehen zu können und ein mehrskaliges Ermüdungsmodell zu erhalten. Zur Darstellung der anfänglichen Rissbildung und des frühen Wachstumsstadium, wurde ein hierarchischer Ansatz angewandt und die Verweildauer in diesen Stadien durch ein Repräsentatives Volumen Element (RVE) unter Einbezug der lokalen zyklischen Mikroplastizität abgeschätzt. Die Stufe des Kurzrisswachstums setzt sich aus dem mikrostrukturellen Kurzriss- (MSC) und dem physikalischen Kurzrisswachstum (PSC) zusammen. Für das Kurzrisswachstum wurde die zyklische Verschiebung an der Öffnung an der Rissspitze ($\Delta CTOD$) als die treibende Kraft identifiziert. Von dieser Beziehung ausgehend wurde die Auswirkungen der mikrostukturellen Ausprägung des Werkstoffs auf das zyklische Kurzrisswachstum explizit bestimmt. Zur Berechnung der Zeitdauer des Langzeitrisswachstums wurde ein Schadensakkumulationskonzept implementiert. Unter der Annahme, dass die Ausbreitungsrate des Langrisswachstums aus den Kennwerten zur Kurzeitfestigkeit (LCF) abgeleitet werden kann, welche sich aus den Ergebnissen von zyklischen Zugversuchen interpolieren lassen [3], konnten Zeitsparend die Parameter zur Lebensdauerberechnung bestimmt werden.

Der wichtigste Beitrag dieser Studie besteht in der Simulation und Modellierung der Einflüsse von Karbiden auf die Mechanismen der Ermüdungsrißausbreitung auf drei unterschiedlichen Längenskalen. Das vorgeschlagene Modell wird als starkes Werkzeug angesehen, um sowohl die Lebensdauer von Werkzeugstählen als auch von partikelverstärkten Kompositwerkstoffen und anderen heterogenen Materialien vorherzusagen. Auf Basis des in der Studie vorgestellten Modells kann eine Optimierung der mikrostrukturellen Eigenschaften der o.g. Materialklassen durchgeführt werden. Die Optimierung kann in einer Erhöhung der Betriebszeiten resultieren.

ABSTRACT

Title: Multi-scale model for fatigue in carbide-rich tool steel

Carbide-rich tool steel is most commonly used not in the tooling industry, but also in engine parts, e.g. springs, bearings, diesel injections, connecting rods etc.. Components made from this kind of material are often subjected to cyclic mechanical stresses. Fatigue is important as it occupies the largest cause of failure in metal, approximately estimated 90% of all metallic failures [1], tool steels are also susceptible to this type of failure. Fatigue resistance of this material strongly depends on the microstructural features including shapes, shape ratio, volume fractions, and distributions of primary and eutectic carbides. Thus, besides loading condition microstructural features are considered as the main factor which influences lifetime of tool components.

It is known that the lifetime prediction of carbide-rich tool steel in alternating applied stress is not an easy task to perform. Therefore, gaining knowledge about the effects of microstructural features on the fatigue behavior of this material is necessary. Subsequently, the main objective of this research is to develop a simple model as well as a computational framework to quantify the influence of these microstructural features on the fatigue behavior of the material in the high cycle fatigue (HCF) regime.

In general, fatigue crack mechanisms can be divided into 3 stages: initial crack formation (crack incubation or nucleation), short crack and long crack growth, which have successfully been established by McDowell [2], in a so-called multistage fatigue model (MSF). To model fatigue behavior of carbide rich tool steel, McDowell's model was modified and developed at three length-scale levels, resulting in a multi-scale fatigue model. For fatigue crack formation and early growth, a hierarchical approach was used, and lifetime of this stage was estimated based on local cyclic micro plasticity within a representative volume element (RVE). The short crack stage consists of microstructurally short crack (MSC) and physically short crack (PSC) growth in which short crack driving force was determined from the process zone at the crack tip, so-called cyclic crack tip opening displacement ($\Delta CTOD$). From this relation, the effects of microstructural features on the cyclic short crack growth were explicitly identified. For long crack growth, an accumulated fatigue damage concept was implemented to calculate the lifetime of this stage. Based on that relation, the long crack growth rate was easily derived from low cycle fatigue (LCF) properties because it is believed that LCF test is easy to calibrate and it may be interpolated from monotonic tensile test [3], which results in saving time and cost for fatigue prediction.

The most important contributions of this study are to simulate and model the influence of carbides on three different length scales of fatigue crack mechanisms in tool steels. The proposed model is considered as a powerful tool for lifetime prediction not only in tool steels, but also in particle reinforced composites and other heterogeneous

materials. Moreover, optimization process on microstructural features can be done based on the results of this study. Consequently, the in-service life of materials may be improved.

Keywords: High Cycle Fatigue, Tool Steel, Fatigue Crack Incubation, Short Crack, Long Crack, Multi-scale Fatigue Model.

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DECLARATION

I, Giang Ngoc Anh, declare that this thesis titled: 'Multi-scale model for fatigue of carbide-rich tool steel' and the work presented in it are my own. I confirm that:

- This work was done mainly while in candidature for a research degree at IWM-RWTH Aachen and no portion of the work referred to in this thesis has previously been submitted for a degree or any other qualification at other university or at any other institution.
- Any material previously published or written by another person that is always clearly attributed in the text.
- I have acknowledged all main sources of help. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

Date: October 30, 2014

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*Giang Ngoc Anh
Aachen, Germany, October 30, 2014*

NOMENCLATURES

1. List of Abbreviations

| | |
|------------------|---|
| AISI | American Iron and Steel Institute |
| BCC | Body Center Cubic |
| CTOD | Crack Tip Opening Displacement |
| DIN | Deutsches Institut für Normung |
| DS | Directionally Solidified |
| expt | Experiment |
| ε -N | Strain - Number of cycles |
| FEA | Finite Element Analysis |
| FEM | Finite Element Method |
| FDZ | Fatigue Damage Zone |
| FS | Fatemi-Socie parameter |
| GBF | Granular Bright Face |
| GO | Grain Orientation |
| GS | Grain Size |
| HCF | High Cycle Fatigue |
| HIP | Hot Isostatic Pressing |
| HP | Hard Phase |
| HSS | High Speed Steels/Carbide-rich Tool Steel |
| HWS | Hot Work tool Steels |
| INC | Incubation |
| JIS | Japanese Industrial Standard |
| LC | Long Crack |
| LCF | Low Cycle Fatigue |
| LEFM | Linear Elastic Fracture Mechanics |
| LENS | Laser Engineered Net Shaping |
| M3:2 | M3 class 2 |
| MMC | Metal Matrix Composite |
| MP | Matrix Phase |
| Mo-alloys | Molybdenum alloys |
| MSC | Microstructurally Small/Short Crack |
| MSF | Multi-Stage Fatigue |
| PBCs | Periodic Boundary Conditions |
| PSBs | Persistent Slip Bands |
| PEEQ | Equivalent Plastic Strain |

| | |
|------|-------------------------------|
| PM | Powder Metallurgy |
| PSC | Physically Small/Short Crack |
| RVE | Representative Volume Element |
| sc | short crack |
| SCG | Short Crack Growth |
| SEM | Scanning Electron Microscope |
| SF | Spray Forming |
| S-N | Stress - Number of cycles |
| SWT | Smith-Watson-Topper parameter |
| th | threshold |
| VHCF | Very High Cycle Fatigue |
| w.% | Weight percentage |

2. Symbol

| Symbol | Description |
|-------------------------|--------------------------------------|
| a | crack length |
| a_0 | initial crack length/intrinsic crack |
| a_D | critical defect |
| a_{inc} | incubation crack length |
| a_f | final crack length |
| $A_{carbide}$ | the area of a carbide |
| $A_{local_plas_zone}$ | local plastic zone area |
| A_p | coefficient of Paris's law |
| a_{sc} | short crack length |
| b | fatigue strength exponent |
| c | fatigue ductility exponent |
| \bar{c} | kinematic modulus |
| \bar{C} | power law coefficient |
| $C_{I,II}$ | coefficient of short crack growth |
| C_p | coefficient of Paris' law |
| c_s | surface crack |
| C_{sc} | coefficient of short crack law |
| C_ϵ | adhesive coefficient |
| D | square root of carbide area |
| \bar{D} | plasticity damage parameter |
| da/dN | crack growth rate |
| $da/dN _{msc/msc}$ | msc/msc crack growth rate |
| $da/dN _{sc}$ | short crack growth rate |
| $da/dN _{lc}$ | long crack growth rate |
| D_c | size of cyclic plastic zone |
| \bar{D} | damage indicator |
| \bar{D}_c | critical damage indicator |
| DCR | cluster diameter of carbides |

| | |
|---------------|--|
| D_{cr} | critical size defect |
| D_{FDZ} | fatigue damage plastic zone or process zone |
| d | diameter of a specimen |
| D_m | size of monotonic plastic zone |
| d_g | autensitic grain size |
| d_o | characteristic length of dislocations |
| \bar{D}_p | average grain size |
| D_p | equivalent diameter of carbide |
| d_s | strongest microstructural barrier |
| D_{sp} | diameter of spherical carbide clusters |
| \dot{D} | rate of plasticity damage parameter |
| E | Young's modulus |
| F | strengthening ratio |
| \bar{F} | coefficient of the cumulative distribution function |
| f^* | calculated void volume fraction |
| f | actual void volume fraction |
| f_c | void volume fraction at failure |
| f_{crit} | critical void volume fraction |
| h | length/height of carbide |
| \bar{k} | exponent of the cumulative distribution function |
| k_o | power law exponent |
| K^* | interaction coefficient of tension and torsion |
| K | hardening coefficient |
| K' | cyclic hardening coefficient |
| K_{max} | maximum applied stress intensity factor |
| K_{min} | minimum applied stress intensity factor |
| K_{op} | crack opening stress intensity factor |
| K_y | coefficient of Hall-Petch's equation |
| L | dimension of RVE model |
| l_s | characteristic length of microstructural features |
| m_p | exponent of Paris' law |
| m_{sc} | exponent of short crack law |
| \bar{n} | a certain number of critical defects |
| n | hardening exponent |
| n' | cyclic strain hardening exponent |
| N_f | number of cycles to failure |
| N_{inc} | number of cycles to fatigue crack incubation |
| N_∞ | infinity life |
| $N_{smc/psc}$ | number of cycles to microstructurally/physically small crack |
| N_{sc} | number of cycles to short crack growth |
| N_{lc} | number of cycles to long crack |
| N_o | number of cycles to failure to limit of LCF |
| N_T | total lifetime |
| N_{T^*} | transition lifetime |
| N^* | number of cycles to crack though the process zone |

| | |
|--------------------------------|---|
| P_f | fracture probability |
| r | polar position from crack tip |
| \bar{r} | dynamic rate of back stress tensor |
| R | stress/strain ratio |
| R_ϵ | strain ratio |
| R_σ^* | stress/strain ratio in the process zone |
| $s = h/w$ | shape ratio (length/width) |
| S | area of fatigue damage zone |
| S_p | mean free path of carbides |
| S_{ij} | deviatoric part of stress tensor |
| s_{sp} | striation spacing |
| V | unit volume |
| V_p | volume fraction of carbide |
| V_m | volume fraction of martensitic steel |
| U | load ratio function |
| $U_x _{appl.}$ | applied displacement in X direction |
| $U_y _{appl.}$ | applied displacement in Y direction |
| u_y | applied displacement in Y direction |
| Y | geometry correction factor |
| w | width (diameter) of carbide |
| α_{ij} | back stress tensor |
| α | fatigue crack incubation exponent |
| α'_{ij} | deviatoric part of back stress tensor |
| $\beta = -(b + c)$ | fatigue power law exponent |
| χ | coefficient of msc/msc crack growth |
| δ | distance to free surface |
| Δa | crack length increment |
| $\Delta CTOD$ | crack tip opening displacement |
| $\Delta CTOD_{th}$ | crack tip opening displacement threshold |
| $\Delta \bar{D}$ | increment of plasticity damage parameter |
| $\Delta \bar{D}_S$ | increment of plasticity damage over stabilization cycle |
| ΔJ | cyclic J integral |
| ΔK | stress intensity factor |
| ΔK_{eff} | effective stress intensity |
| $\Delta \bar{K}_{eff}$ | effective stress intensity in bulk composite |
| $\Delta K_{eff,th}$ | effective threshold stress intensity factor |
| ΔK_{th} | threshold stress intensity factor |
| $\Delta K_{th,lc}$ | threshold stress intensity factor for long crack |
| $\Delta K_{th,R}$ | threshold stress intensity factor as a function of load ratio |
| $\Delta K_{th,sc}$ | threshold stress intensity factor for short crack |
| ΔK_T | threshold stress intensity for transition region |
| $f(\sigma_{ij} - \alpha_{ij})$ | von Mises flow surface |
| $\varepsilon_{pl,n}$ | normal plastic strain on a critical plane |
| $\Delta \varepsilon_{pl,ij}^I$ | different plastic strain on the critical plane |
| $\Delta \varepsilon_{pl}$ | plastic strain range |

| | |
|---|---|
| $\gamma_{pl,n}$ | plastic shear strain on a critical plane |
| $\Delta\Gamma$ | Fatemi-Socie parameter |
| Γ_{ij} | boundary of RVE (i,j =1,2) |
| $\frac{\Delta\gamma_{max}^{p*}}{2}$ | maximum cyclic plastic shear strain range |
| $\frac{\Delta\gamma_{max}^{p*}}{2} \Big _{macro}$ | remote applied plastic shear strain range |
| ϵ_a | remote strain amplitude |
| ϵ_C | equivalent plastic strain at initial damage |
| ϵ'_f | fatigue ductility coefficient |
| ϵ_R | equivalent plastic strain at final failure |
| $\dot{\epsilon}_{eq}^{pl}$ | rate of equivalent plastic strain |
| $\epsilon_{pl,ij}$ | plastic strain tensor |
| ϵ_{pl}^* | plastic strain over fatigue damage zone |
| ϵ_{eq}^{pl} | equivalent plastic strain |
| φ | angle of critical plane |
| $\varphi(f)$ | function of void volume fraction |
| θ | polar angle in the cyclic plastic zone |
| ℓ | square root of local plastic zone area |
| κ | flow stress exponent |
| $\bar{\lambda}_c$ | an expected number of critical defects |
| λ | correction factor |
| η | stress triaxiality |
| ω | frequency of applied loading |
| Ω | volume of local plastic zone |
| Π | potential energy per unit volume |
| Π_{int} | interaction energy per unit volume |
| Φ | von Mises plastic flow |
| ψ | correction factor of cyclic J integral |
| ρ_c | radius of crack blunting |
| $\Delta\sigma$ | stress range |
| $\Delta\sigma_o$ | stress threshold range |
| $\Delta\sigma_{ij}^I$ | different stress on the critical plane |
| σ_a | remote stress amplitude |
| σ_{eq} | equivalent stress (von Mises stress) |
| σ'_f | fatigue strength coefficient |
| $\sigma_f^{carbide}$ | fracture strength of a carbide |
| σ_f | strength (flow stress) of a fiber/particle |
| σ_H | hydrostatic stress |
| σ_I | instantaneous principle stress |
| σ_{ij} | stress tensor |
| σ_∞ | coefficient of flow stress |
| σ_{iso} | flow stress |
| σ_{max} | maximum stress in the process zone |
| σ_{min} | minimum stress in the process zone |

| | |
|--------------------------|---|
| $\sigma_{max}^{carbide}$ | maximum stress within a carbide |
| σ_m | strength (flow stress) of matrix phase |
| σ_m^* | mean stress within process zone |
| σ_n^{max} | normal stress on a critical plane |
| $\Delta\sigma_{ult}$ | range of fracture strength |
| $\Delta\sigma_{ult}'$ | range of fracture strength |
| $\Delta\hat{\sigma}$ | equivalent applied stress range |
| $\sigma_{p,C}$ | strength of bulk composite |
| $\sigma_{p,M}$ | strength of the matrix phase-soft phase |
| $\sigma'_{p,C}$ | cyclic yield strength of composite material |
| σ^o | initial yield strength of material |
| σ_y | yield strength of bulk material |
| σ'_y | cyclic yield strength of bulk material |
| $\sigma'_{p,M}$ | cyclic yield strength of the matrix phase |
| σ_{ult} | ultimate tensile stress |