

Robust Multi-stage Nonlinear Model Predictive Control

Zur Erlangung des akademischen Grades eines

Dr.-Ing.

von der Fakultät Bio- und Chemieingenieurwesen
der Technischen Universität Dortmund
genehmigte Dissertation

vorgelegt von

Sergio Lucia, M.Sc.

aus

Zaragoza, Spanien

Tag der mündlichen Prüfung: 10.12.2014

1. Gutachter: Prof. Dr. Sebastian Engell
2. Gutachter: Prof. Dr. Lorenz T. Biegler

Dortmund 2015

Schriftenreihe des Lehrstuhls für
Systemdynamik und Prozessführung
herausgegeben von Prof. Dr.-Ing. Sebastian Engell

Band 4/2015

Sergio Lucia

**Robust Multi-stage Nonlinear
Model Predictive Control**

D 290 (Diss. Technische Universität Dortmund)

Shaker Verlag
Aachen 2015

Bibliographic information published by the Deutsche Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Dortmund, Technische Univ., Diss., 2014

Copyright Shaker Verlag 2015

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 978-3-8440-3609-1

ISSN 1867-9498

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: www.shaker.de • e-mail: info@shaker.de

Acknowledgements

First and foremost I would like to thank Prof. Sebastian Engell for giving me the opportunity of writing my PhD thesis under his supervision. It has been for me a very motivating and enriching experience and I am deeply grateful for his support and discussions during these four years, including the pressure that helped me to improve and try my best. I am also very grateful to Prof. Lorenz T. Biegler for agreeing to be the external referee of my thesis and for his very valuable feedback.

The work presented in this thesis would not have been possible without the help and the collaboration of my colleagues at the Group of Process Dynamics and Operations of the TU Dortmund. I would like to thank all the colleagues, secretaries and also the students that I supervised. Especially, I would like to thank Radoslav Paulen for all the work that we have done together, which has been very motivating for me, for proof-reading this thesis, and for teaching me my first words in Slovak while spending long hours of research together. I would like to thank also Daniel Haßkerl mainly for all the time that we have spent together during the organization of the International Summer Program of the TU Dortmund. I am also very grateful to all the colleagues which I have collaborated with. Many thanks especially to Sankaranarayanan Subramanian, Tiago Finkler, Ehsan Nabati, Christian Schoppmeyer, Lars Simora and Malte Behrens.

During my time in Dortmund I had the opportunity to participate in several projects and I would like to thank my colleagues from other universities for the productive and motivating collaborations. I am especially grateful to Joel Andersson for all the work together regarding CasADI and also to Daniel Limon for the joint work on the stability of the multi-stage approach.

I am deeply grateful to Alba for happily sharing with me these years in Germany, and for her continuous support, love and understanding. Finally, I would like to thank all my family, especially my parents and my brothers for being an endless source of support and love.

Abstract

Model Predictive Control (MPC) has become one of the most popular control techniques in the process industry mainly because of its ability to deal with multiple-input-multiple-output plants and with constraints. However, its performance can deteriorate in the presence of model uncertainties and disturbances. In the last years, the development of robust MPC techniques has been widely discussed, but these were rarely applied in practice due to their conservativeness or their computational complexity.

This thesis presents multi-stage nonlinear model predictive control (multi-stage NMPC) as a promising non-conservative robust NMPC control scheme, which is applicable in real-time. The approach is based on the representation of the evolution of the uncertainty by a scenario tree. It leads to non-conservative robust control of the plant because it takes into account explicitly that new information (usually present as measurements) will become available at future time steps and that the future control inputs can be adapted accordingly, acting as recourse variables.

Different aspects of the proposed multi-stage NMPC scheme are studied in detail in this thesis. Firstly, the approach is analyzed from a control theory point of view, including a formulation that guarantees stability and constraint satisfaction. Secondly, an efficient implementation is described, which is necessary to deal with one of the challenges of the presented method: The size of the resulting optimization problems. Thirdly, novel algorithms and modifications are proposed to enhance its performance and capabilities.

The method is evaluated using examples from the chemical engineering field. Several simulations and real experiments presented in this thesis show that multi-stage NMPC is a promising strategy for the optimizing control of uncertain nonlinear systems subject to hard constraints. It is also shown that multi-stage NMPC performs better than standard NMPC and better than other robust NMPC approaches presented in the literature while still being implementable in real-time.

Zusammenfassung

Modellprädiktive Regelung (kurz: MPC) ist eine der populärsten Methoden zur Regelung von Anlagen in der chemischen Industrie. Dies beruht auf der Möglichkeit, Mehrgrößensysteme mit Beschränkungen zu behandeln. Ungenauigkeiten im mathematischen Modell und äußere Einflüsse führen allerdings zu einer Verschlechterung der Regelgüte. Um die genannte Probleme zu vermeiden, wurden in den letzten Jahren verschiedene robuste MPC-Ansätze untersucht. Diese Ansätze finden aber in der Praxis selten Anwendung, weil sie sehr konservative Lösungen liefern oder ihre numerische Komplexität zu hoch ist.

In dieser Dissertation wird die Methode des mehrstufigen nichtlinearen MPC als ein vielversprechender robuster und nicht konservativer MPC-Ansatz vorgestellt. Die Methode basiert auf der Beschreibung der Unsicherheiten als Szenarienbaum. Dies führt zu einer nicht konservativen und robusten Regelung der Strecke, weil explizit berücksichtigt wird, dass in der Zukunft neue Informationen (oft in der Form von Messungen) zur Verfügung stehen. Die zukünftigen Stellgrößen können dann entsprechend angepasst werden und haben die Wirkung von *recourse* Variablen.

Verschiedene Aspekte des vorgestellten mehrstufigen nichtlinearen MPC-Ansatzes werden in dieser Dissertation ausführlich untersucht. Zunächst wird eine Analyse aus der Perspektive der Regelungstheorie vorgestellt, einschließlich einer Formulierung, welche die Stabilität und die Einhaltung der Nebenbedingungen garantieren kann. Im nächsten Schritt wird eine effiziente Implementierung beschrieben, die notwendig ist, um die wichtigste Herausforderung des Ansatzes zu bewältigen: die Größe des zu lösenden Optimierungsproblems. Im Anschluss daran werden neue Algorithmen und Ergänzungen vorgeschlagen, um das Potenzial des Ansatzes zu erhöhen.

Die vorgestellte Methode wird mit Hilfe von Beispielen aus der Verfahrenstechnik evaluiert. Mehrere Simulationen und Experimente werden in dieser Dissertation gezeigt. Die Ergebnisse deuten darauf hin, dass die Methode des mehrstufigen nichtlinearen MPC eine vielversprechende Strategie für die optimierungsbasierte Regelung unsicherer Systeme mit Beschränkungen ist. Im Vergleich zu Standard MPC oder anderen robusten MPC Methoden, weist der vorgestellte Ansatz eine bessere Regelgüte auf, während die gleichzeitig in Echtzeit realisierbar ist.

Contents

1	Introduction	1
1.1	Motivation	1
1.2	Scope of the Thesis	2
1.3	Structure and Contribution of the Thesis	2
I	Theoretical Foundations	7
2	Optimization-based Robust Control	9
2.1	Optimal Control	9
2.2	Optimization Under Uncertainty	10
2.3	Robust Model Predictive Control	14
2.4	Real-Time Optimization	17
2.5	Discussion	19
3	Multi-stage NMPC as a General Framework for Robust NMPC	21
3.1	Multi-stage Nonlinear Model Predictive Control	21
3.1.1	The Robust Horizon Assumption	25
3.2	Open-loop Robust Nonlinear Model Predictive Control	26
3.3	The Importance of Recourse	27

3.4	Robust Model Predictive Control with Affine Policies	29
4	Stability Properties of Multi-stage NMPC	31
4.1	Input-to-State Stability of Multi-stage NMPC	34
4.1.1	Notation and Basic Definitions	34
4.1.2	Input-to-State Practical Stability of Multi-stage NMPC	36
4.1.3	Achieving Input-to-State Stability of Multi-stage NMPC	40
4.2	Multi-stage NMPC with Robust Horizon	40
4.3	Discussion	43
II	Efficient Implementation and Solution	45
5	Formulation and Solution of Dynamic Optimization Problems	47
5.1	Discretization Methods	47
5.1.1	Sequential Approach	48
5.1.2	Simultaneous Approach	48
5.2	Calculation of the Derivatives	53
5.3	Implementation of Multi-stage NMPC	58
6	DO-MPC: An Environment for the Easy Development of NMPC Solutions	61
6.1	A Modular NMPC Development Environment	62
6.2	DO-MPC: An Environment for an Easy, Modular, Robust and Efficient Development of NMPC	64
7	Solution via Scenario Decomposition Techniques	69
7.1	Decomposition Approaches for Multi-stage Optimization	70

7.1.1	Scenario Decomposition	70
7.1.2	Reducing the Number of Iterations	72
7.1.3	Discussion	73
III	Simulation and Experimental Results	75
8	Multi-stage NMPC of Polymerization Processes	77
8.1	The Chylla-Haase Polymerization Reactor	78
8.1.1	Standard, Min-max and Multi-stage Economic NMPC of the Polymerization Reactor Example with Full Feedback Information	82
8.1.2	Multi-stage NMPC with State and Parameter Estimation	92
8.2	An Industrial Polymerization Reactor	98
8.2.1	Tracking NMPC vs. Multi-stage Economic NMPC of a Polymerization Reactor under Uncertainty	105
8.2.2	Comparison of Robust Economic NMPC formulations for a Polymerization Reactor under Uncertainty	110
8.2.3	Results with Scenario Decomposition	112
8.3	Discussion	118
9	Multi-stage NMPC with Guaranteed Stability	121
9.1	Illustrative Example	121
9.2	Calculation of the Terminal Ingredients	122
9.3	Results	123
10	Experimental Results of Multi-stage NMPC of a Laboratory Plant	129
10.1	A Continuous Stirred Tank Reactor	129
10.2	Simulation and Experimental Results	134

IV Extended Algorithms and Enhancements	143
11 Multi-stage NMPC with Reduced Variability	145
12 Multi-stage NMPC with Verified Robust Constraint Satisfaction	151
12.1 Computing the Reachable Sets	152
12.1.1 Interval Bounds	153
12.1.2 Ellipsoidal-based Bounds	154
12.2 Verified Robust Constraint Satisfaction using Multi-stage NMPC	155
12.3 Case Study	160
13 Multi-stage NMPC with Reduction of the Uncertainty	165
13.1 Robust Optimal Dynamic Experiment Design	166
13.2 Guaranteed Parameter Estimation	168
13.3 Proposed Algorithm	170
13.4 Case study	173
13.5 Using the Correct Sensitivity Information	178
V Summary, Conclusions, and Future Work	183
14 Conclusions and Future Work	185
14.1 Conclusions and Guidelines for the Use of Multi-stage NMPC	185
14.2 When and How to Use Multi-stage NMPC?	187
14.2.1 When to Use Multi-stage NMPC	187
14.2.2 How to Use Multi-stage NMPC	189
14.3 Future Work	190

A Multi-stage NMPC for Setpoint Tracking	193
A.1 Case Study	193
A.2 Setpoint Tracking under uncertainty	195
B List of Symbols	203