

# **Linear Time-Periodic Control and Analysis of Helicopter Air Resonance**

A thesis accepted by the Faculty of Aerospace Engineering and Geodesy  
of the University of Stuttgart in partial fulfillment of the requirements  
for the degree of Doctor of Engineering Sciences (Dr.-Ing.)

by

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from Rennes, France

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|-------------------|-------------------------------|
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*À mon Père,  
pour m'avoir toujours offert la chance  
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