



# Summability Methods for the Inversion of the Spherical Mean Operator

**Dissertation**

To Fulfill the  
Requirements for the Degree of  
Doctor of Science

Submitted to the  
Faculty of Mathematics and Computer Science  
of the  
University of Osnabrück

by Ruben Seyfried



Berichte aus der Mathematik

**Ruben Seyfried**

**Summability Methods for the Inversion  
of the Spherical Mean Operator**

Shaker Verlag  
Aachen 2014

**Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Osnabrück, Univ., Diss., 2014

Copyright Shaker Verlag 2014

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 978-3-8440-3000-6

ISSN 0945-0882

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: [www.shaker.de](http://www.shaker.de) • e-mail: [info@shaker.de](mailto:info@shaker.de)

## Acknowledgment

Above all I wish to thank my advisors Dr. Frank Filbir and Prof. Dr. Stefan Kunis for their guidance. Their encouragement and support were invaluable in the creation of this thesis and their approachability, candor and genuine desire to help extended far beyond the limits of the task at hand and provided me with a constructive and positive working environment. I can only say: it has been a pleasure and a privilege knowing and working with you. Thank you for all that you do and have done.

Further thanks go to Prof. Dr. Markus Haltmeier for his assessment as well as his council and advice. Additionally I wish to thank Marcus Ansorg, Martin Ehler, Andreas Weinmann, and Markus Tannenholz, all of whom gave generously of their time at various stages of the process and were able to offer many helpful suggestions.

While I regret it is not possible to list them all here by name, thanks to all the people who encouraged and supported along the way, especially to my great co-workers, my beloved family, and of course my friends.



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Photoacoustic Tomography</b>	<b>7</b>
2.1	Mathematical Model . . . . .	7
2.2	Variation to the Model in Two Dimensions . . . . .	11
2.3	The Spherical Mean Operator . . . . .	12
<b>3</b>	<b>Summability Methods</b>	<b>17</b>
3.1	General Considerations . . . . .	17
3.2	Review for the Classical Radon Transform . . . . .	24
3.3	Application to the Spherical Mean Operator . . . . .	27
3.3.1	The Two-Dimensional Case . . . . .	31
3.3.2	The Three-Dimensional Case . . . . .	44
3.3.3	Relation to the Wave Equation . . . . .	49
3.3.4	Inversion Formulae . . . . .	51
3.4	Degree of Approximation . . . . .	53
<b>4</b>	<b>Effective Discretization</b>	<b>61</b>
4.1	Circular Means . . . . .	61
4.2	Spherical Means . . . . .	65
4.2.1	Parameter Choice and Computational Complexity . . . . .	73
4.2.2	Alternative Discretization Strategies . . . . .	79
<b>5</b>	<b>Numerical Results</b>	<b>83</b>
5.1	Circular Means . . . . .	83
5.2	Spherical Means . . . . .	85
5.3	Degree of Approximation . . . . .	87
<b>6</b>	<b>Related Work</b>	<b>93</b>
6.1	Exact Inversion Formulae . . . . .	94
6.2	Approximate Inverse and Mollification . . . . .	97
6.3	Numerical Approaches . . . . .	99
6.4	Regularization by Summability Methods . . . . .	101
	<b>Bibliography</b>	<b>103</b>