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Operator Triplets In Applied Mathematics

Integral Transforms and Fractional Integrals

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Theory of integral transforms is an important part of applied mathematics and signal processing. An integral transform can be defined by a pair of orthogonal integral operators, whose kernels can be represented in terms of functions of two variables. Usually kernels of these operators are expressed by means of regular functions and based on complete orthogonal systems. The integral transforms yield a power tool for investigation of differential, difference and integral equations.

Fractional integrals also consist of pair of orthogonal operators, however their kernels involve weak-singular generalised functions of power type. Abel integral equations form foundation of theory of the fractional integrals.

There is an interesting feature of integral operators: various integral operators can be combined among themselves by means of operator product. These combinations of integral transforms open new ways and opportunities for solving of problems of applied mathematics. For example, the fractional integrals can connect two integral transforms with different regular kernels. Mathematically this relation can be expressed by means of discontinuous integrals. Thus the well-known discontinuous Weber integrals can be treated as a relation of Fourier transform and Hankel transform by means of Abel fractional integrals.

In this work we introduce under consideration a new mathematical structure: operator triplet (triplet of orthogonal integral operators or triplet of integral transforms). We define operator triplet as a combination of three pair of orthogonal integral operators. Generally an operator triplet is a closed system of mathematical structures, which consists of three orthogonality relations of integral transforms and system of integral representations for kernels of orthogonal systems.

An important group of triplets form combinations of two integral transforms with regular kernels and pair of fractional integrals (triplet of integral transforms). This triplet contains two orthogonality relations of integral transforms, pair of fractional integrals, system of discontinuous integrals and system of integral representations for regular kernels of integral transforms. Thus we can say, that the Weber integrals based on triplet Fourier-Hankel-Abel. An important role play in this development Mehler integrals, which correspond to triplet Legendre-Fourier-Abel and Sonin integrals, based on triplet Hankel-Hankel-Abel.

The principal purpose of this work is to find basic regularities of creation of different combinations of orthogonal systems, to construct sequential theory of operator triplets. In this work we present general theory of operator triplets and construct examples of triplets with different combinations of kernels. Applications of method of triplets for solving of integral equations of the first kind and for dual integral equations are given. Generally technique of triplets form a power tool for investigation of problems of applied mathematics and signal processing.