The Two-Dimensional, Rectangular, Guillotineable-Layout Cutting Problem with Defects

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For my wife Sara, with much love and many thanks

Abstract

This thesis deals with a two-dimensional cutting problem in which small rectangular items of given types are to be cut from a rectangular large object which contains several defects. It is assumed that the number of pieces of each small item type, which can be cut from the large object, is not limited. In addition, all cuts are restricted to be of the guillotine-type and the number of stages, which are necessary to perform all cuts, can either be limited or not. Furthermore, no small item must overlap with a defective region. The objective is to maximize the value of the cut small items.

In the presence of defects, the definition of discretization sets and the identification of duplicated patterns are revisited. For the exact solution of the above-described problem, dynamic programming algorithms are presented. Moreover, the computational complexity of the algorithms is analyzed and the factors which affect the running time of the algorithms are identified.

The exact algorithms are able to solve the problem instances of small and medium sizes. For large problem instances for which optimal solutions cannot be computed efficiently, the dynamic programming algorithms are modified into algorithms in which an adaptation of a variable beam search approach equipped with several heuristic rules is applied.

The proposed algorithms are evaluated by means of a series of detailed numerical experiments which are performed on problem instances extracted from the literature, as well as on randomly generated instances. The experiments do not only illustrate how the proposed method can identify optimal solutions of the test problem instances, but they also explain why already existing methods fail to do so. Furthermore, the computational results indicate that the exact and heuristic recursive-based methods are able to overcome efficiently some structural and computational limitations for solving problem instances of realistic sizes.

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List of Symbols

B	set of nodes
BV	best variable value
CP	characteristic function
$Depth(P)$	depth bound for pure plates
$Depth(C)$	depth bound for contaminated plates
$D_{x,y}$	defective area of a contaminated plate $N = (ox, oy, x, y)$
E	beam width
$F(ox, oy, x, y)$	optimal objective function value for a contaminated plate (ox, oy, x, y)
$F(x, y)$	optimal objective function value for a pure plate (x, y)
$F^1(ox, oy, x, y)$	objective function value computed by the first heuristic algorithm for a contaminated plate (ox, oy, x, y)
$F^1(x, y)$	objective function value computed by the first heuristic algorithm for a pure plate (x, y)
$F^2(ox, oy, x, y)$	objective function value computed by the second heuristic algorithm for a contaminated plate (ox, oy, x, y)
$F^2(x, y)$	objective function value computed by the second heuristic algorithm for a pure plate (x, y)
$F^3(ox, oy, x, y)$	objective function value computed by the third heuristic algorithm for a contaminated plate (ox, oy, x, y)
$F^3(x, y)$	objective function value computed by the third heuristic algorithm for a pure plate (x, y)
$F_k(ox, oy, x, y)$	optimal objective function value for the *k*-stage cutting pattern of a contaminated plate (ox, oy, x, y), where the cut at the first stage is a vertical one
$F_k(x, y)$	optimal objective function value for the *k*-stage cutting pattern of a pure plate (x, y), where the cut at the first stage is a vertical one
$G_k(ox, oy, x, y)$	optimal objective function value for the *k*-stage cutting pattern of a contaminated plate (ox, oy, x, y), where the cut at the first stage is a horizontal one
$G_k(x, y)$	optimal objective function value for the *k*-stage cutting pattern of a pure plate (x, y), where the cut at the first stage is a horizontal one
$g_s(x, y)$	maximal objective function value which corresponds to the allocation of the most valuable small item type to plate (x, y)
$g(x, y)$	maximal objective function value of all homogeneous cutting patterns of a plate (x, y)
$H_{m+n}(z_y)$	horizontal tableau for the computation of horizontal discretization set
i	defect type
$innermax$	temporary maximum value
$innermin$	temporary minimum value

j	item type
k	number of stages
L	set of all possible positions for vertical cuts
L_0	length of the large object
$\overline{L_0}$	vertical discretization sets for the large object (SLOPP)
$\lvert\overline{L_0}\rvert$	number of elements within the vertical discretization set (SLOPP)
$LB(ox, oy, x, y)$	lower bound for the objective function value of the optimal cutting pattern for a contaminated plate $N = (ox, oy, x, y)$
$LB(x, y)$	lower bound for the objective function value of the optimal cutting pattern for a pure plate $N = (x, y)$
l_i^d	length of defect i
$list(ox, oy, x, y)$	list of elite divisions related to a contaminated plate $N(x, y)$
$list(x, y)$	list of elite divisions related to a pure plate $N(x, y)$
l_j^s	length of small item type j
m	number of small item types
N	intermediate plate
O	complexity function
(ox, oy)	location of an intermediate plate
$PositiveInfinity$	a big number
$p(x)$	element nearest to x in the vertical discretization set
$q(y)$	element nearest to y in the horizontal discretization set
T_B	average computing time obtained by an exact algorithm
$temp$	temporary variable
T_H	average computing time obtained by a heuristic algorithm
$UB(ox, oy, x, y)$	upper bound for the objective function value of the optimal cutting pattern for a contaminated plate $N = (ox, oy, x, y)$
$UB(x, y)$	upper bound for the objective function value of the optimal cutting pattern for a pure plate $N = (x, y)$
v_j	value of small item type j
$V_{m+n}(z_x)$	vertical tableau for the computation of vertical discretization set
W	set of all possible positions for horizontal cuts
W_0	width of the large object
$\overline{W_0}$	horizontal discretization sets for the large object (SLOPP)
$\lvert\overline{W_0}\rvert$	number of elements within the horizontal discretization set (SLOPP)
w_i^d	width of defect i
w_j^s	width of small item type j
(x, y)	dimensions of an intermediate plate
$\overline{x}$	vertical discretization set for (x, y) (SLOPP)

$\bar{y}$	horizontal discretization set for (x, y) (SLOPP)
Z_B	average objective function value obtained by an exact algorithm
Z_H	average objective function value obtained by a heuristic algorithm
z_x	position of a vertical cut
z_y	position of a horizontal cut
α_j	non-negative variable attached to the length of small item type *j*
β_j	non-negative variable attached to the width of small item type *j*
δ_i	non-negative variable attached to the length of defect *i*
θ^c	parameter used in the third heuristic algorithm for contaminated plates
θ^p	parameter used in the third heuristic algorithm for pure plates
λ_x^c	vertical parameter used in the first heuristic algorithm for contaminated plates
λ_x^p	vertical parameter used in the first heuristic algorithm for pure plates
λ_y^c	horizontal parameter used in the first heuristic algorithm for contaminated plates
λ_y^p	horizontal parameter used in the first heuristic algorithm for pure plates
μ_i	non-negative variable attached to the width of defect *i*
ρ_l	smallest length of item types
ρ_w	smallest width of item types
ρ^s	parameter used for the generation of small item types
σ	length and width of a quadratic large object
τ_j	number of times item type *j* appears in the large object
φ_l	step function (lower border)
φ_u	step function (upper border)
ω_r^x	non-negative variable for the representation of vertical spaces
ω_r^y	non-negative variable for the representation of horizontal spaces
$\Omega_x^{CH}(x)$	subset of the vertical discretization set for contaminated plates
$\Omega_x(L_0)$	vertical discretization sets for the large object (SLOPP_D)
$\lvert\Omega_x(L_0)\rvert$	number of elements within the vertical discretization set $\Omega_x(L_0)$
$\Omega_x^{PH}(x)$	subset of the vertical discretization set for pure plates
$\Omega_x(x)$	vertical discretization set for (ox, oy, x, y) (SLOPP_D)
$\Omega_y^{CH}(y)$	subset of the horizontal discretization set for contaminated plates
$\Omega_y^{PH}(y)$	subset of the horizontal discretization set for pure plates
$\Omega_y(W_0)$	horizontal discretization sets for the large object (SLOPP_D)
$\lvert\Omega_y(W_0)\rvert$	number of elements within the horizontal discretization set $\Omega_y(W_0)$
$\Omega_y(y)$	horizontal discretization set for (ox, oy, x, y) (SLOPP_D)
∂_l^c	lower bound-based parameter used in the second heuristic algorithm for

	contaminated plates
∂_l^p	lower bound-based parameter used in the second heuristic algorithm for pure plates
∂_u^c,	upper bound-based parameter used in the second heuristic algorithm for contaminated plates
∂_u^p	upper bound-based parameter used in the second heuristic algorithm for pure plates

List of Abbreviations

AAR	average approximation ratio
Av	average
BSC	best strip cutting
com. time	computing time
C&P	cutting and packing
CTR	computational time ratio
DBH1	dynamic programming beam search algorithm 1
DBH2	dynamic programming beam search algorithm 2
DBH3	dynamic programming beam search algorithm 3
DH	depth-first and hill-climbing strategies
DOV	average deviation of the objective function values
DP	discretization procedure
DPC	exact dynamic programming algorithm equipped with all integer positions for guillotine cuts
DPD	exact dynamic programming algorithm with the suggested definition of the discretization sets
FP	filling procedure
GRASP	greedy randomized adaptive search procedure
IZD	instances with a zero deviation
KD	knapsack problems, which are solved by dynamic programming algorithms
NP	non-deterministic polynomial
OFV	objective function value
p	polynomial
SB	simple block
sec	second
SGP	strips generation procedure
SSSCSP	single stock size cutting stock problem
TDC	two-dimensional cutting
1D_U_ SLOPP	one-dimensional, unconstrained, single large object placement problem
2D_ UG_SLOPP	two-dimensional, rectangular, unconstrained, guillotineable-layout, single large object placement problem without defects
2D_UG_SLOPP_D	two-dimensional, rectangular, unconstrained, guillotineable-layout, single large object placement problem with defects