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**Nichtkonforme Gitter für die numerische
Simulation von Aeroakustik- und
Vibroakustikproblemen**

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the Numerical Simulation of Problems from
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Nichtkonforme Gitter für die numerische Simulation von Aeroakustik- und Vibroakustikproblemen (Kurzfassung)

Die Simulation von strömungsinduziertem Schall und Vibrationsschall bringt wegen der großen Skalenunterschiede der beteiligten Felder einige Probleme mit sich. Bei niedrigen Machzahlen sind kleine Strukturen in Strömungen für die Erzeugung von akustischen Störungen verantwortlich, die sich mit Schallgeschwindigkeit ausbreiten. Die charakteristischen akustischen Wellenlängen sind dabei oft groß im Vergleich zu dem Gebiet in dem das Fluid strömt. Mechanische Schwingungen bei Frequenzen bis 1 kHz erzeugen Schall mit Wellenlängen, die mitunter länger als die schwingenden Strukturen sein können. Für beide Phänomene ist der Einsatz von angepassten numerischen Verfahren nötig.

Aus den genannten Argumenten erschließt sich leicht, dass verschiedene Diskretisierungen in den unterschiedlichen Untergebieten des Rechengebiets wünschenswert sind. Die Standard FEM eignet sich jedoch aufgrund ihrer Beschränktheit auf konforme Gitter für diese Probleme nur bedingt. Deswegen wird in dieser Arbeit die Mortar FEM, eine nichtüberlappende Gebietszerlegungsmethode, verwendet. Diese Methode erlaubt nichtkonforme Gitterübergänge an Gebietsgrenzen, was für große Flexibilität bei der Erstellung von Rechenmodellen sorgt. Gitter in Untergebieten können unabhängig voneinander erstellt werden und deren Feinheit kann den physikalischen Gegebenheiten angepasst werden.

Durch das Zulassen von nichtkonformen Gittern müssen die Näherungslösungen in geeigneter Weise über Gebietsgrenzen transferiert werden. Die an den Grenzen auftretenden Koppelintegrale müssen über verschiedene Gitter ausgewertet werden. Es werden daher ausgefeilte Gitterschnittalgorithmen benötigt.

Ähnliche Algorithmen werden auch für die Interpolation von aeroakustischen Quelltermen gebraucht. Die Terme werden zunächst nach der Lighthillschen Analogie auf feinen Gittern für die Strömung berechnet und in einem zweiten Schritt auf relativ grobe Akustikgitter interpoliert.

Die entwickelten Methoden werden auf einige Testfälle angewendet. Für die Aeroakustik werden ein umströmter Zylinder und ein Schneidetonaufbau untersucht. In der Vibroakustik werden das Verhalten eines elektrodynamischen Lautsprechers und piezoelektrische Aktoren zur aktiven Schalldämpfung betrachtet.

Nonmatching Grids for the Numerical Simulation of Problems from Aeroacoustics and Vibroacoustics

(Abstract)

The simulation of flow- and vibration-induced sound generation phenomena poses a number of problems due to large scale disparities. In aeroacoustics for low Mach-number flows small features in a slow flow generate acoustic disturbances which propagate away at the speed of sound. The characteristic acoustic wave lengths are typically large compared to the domain of the flow. Mechanical vibrations at low frequencies of up to about 1 kHz generate sound with wave lengths, which are sometimes even larger than the vibrating structures. Both phenomena require the use of special numerical methods to cope with.

The preceding arguments make the need for different discretizations in different parts of a computational domain quite clear. The standard finite element method (FEM) is not well-suited to overcome these problems, due to its need for conforming meshes. Therefore the Mortar FEM, which is a non-overlapping domain decomposition method and permits non-matching grids at interfaces, gets applied in this thesis. With this method also the generation of computational models becomes much more flexible, since subdomain grids can be generated independently and can therefore be adapted to the physical needs in different parts of the domain.

By allowing non-matching grids the approximate solutions have to be transferred across interfaces in an appropriate way. The coupling integrals arising at these interfaces need to be evaluated with respect to different meshes and therefore sophisticated mesh intersection algorithms have to be implemented.

Similar algorithms have to be developed for the interpolation of aeroacoustic sound sources. The sources get computed from velocity fields according to Lighthill's acoustic analogy on fine computational fluid dynamics (CFD) meshes. Afterwards they get interpolated to relatively coarse acoustic grids in order to reduce the problem sizes.

The developed algorithms get applied to a number of test cases. For aeroacoustics a cylinder in cross-flow and the setup of an edge-tone are investigated. For vibroacoustics the Mortar FEM gets applied to the cases of an electrodynamic loudspeaker and to piezoelectric patches for active noise damping.

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This thesis is the result of my research activities from 2007 until 2011 at the chair of Sensor Technology at the University of Erlangen-Nuremberg and the chair of Applied Mechatronics at the Alps-Adriatic University of Klagenfurt.

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for the aeroacoustic cylinder simulations. The CFD results presented for the edge tone are part of István Vaik's work at the Department of Hydrodynamic Systems at Budapest University of Technology. I would also like to thank Dr. György Paál, the head of this department, for his hospitality during my stay in Budapest and the many fruitful discussions during mutual visits.

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Notations and Abbreviations

In this thesis, scalars are represented by normal letters (b), Cartesian vectors are set in bold-italic letters ($\mathbf{b}$), second order tensors are denoted by bold letters ($\mathbf{b}$) and fourth order tensors by bold letters in square brackets ($[\mathbf{b}]$). Matrices are capital boldface Roman letters ($\mathbf{b}$).

Abbreviations

SAS	Scale adaptive simulation
RHS	Right hand side
POI	Point of interest
FEM	Finite element method
CFD	Computational fluid dynamics
LES	Large eddy simulation
PDE	Partial differential equation
IBVP	Initial boundary value problem
ODE	Ordinary differential equation
HPC	High performance computing

Mathematical conventions

∇b	gradient of scalar-valued function b
$\nabla \cdot \mathbf{b}$	divergence of vector-valued function $\mathbf{b}$
$\nabla \times \mathbf{b}$	curl of vector-valued function $\mathbf{b}$
Δb	laplacian of scalar-valued function b ($\Delta b = \nabla \cdot \nabla b$)
$\int_{\Omega} \square \, d\Omega$	Volume integral over $\square$
$\partial \square / \partial x$	Spatial partial derivative
$\int_{\Gamma} \square \, d\Gamma$	Surface integral over $\square$
$\partial \square / \partial x$	Spatial partial derivative of $\square$
$\square_{i,x}$	Partial derivative of component i of $\square$ in respect to x
$\partial \square / \partial t$	First partial derivative of $\square$ with respect to time
$\partial^2 \square / \partial t^2$	Second partial derivative of $\square$ with respect to time
D/Dt	Substantial or total temporal derivative
$\partial \square / \partial \mathbf{n}$	Directional derivative of $\square$ with respect to $\mathbf{n}$
$\mathbf{I}$	identity matrix
$\square^T$	transpose of $\square$
$\square^{-1}$	inverse of $\square$

Acoustics

c	speed of sound
λ	wave length
ρ_a	density of acoustic fluid
p_a	sound pressure
$\mathbf{v}_a$	acoustic particle velocity
f	frequency
k	wave number
ω	angular frequency

Electromagnetics

$\mathbf{D}$	electric displacement
--------------	-----------------------

ρ_e	free electric volume charge density
$\mathbf{B}$	magnetic flux density
$\mathbf{E}$	electric field intensity
$\mathbf{H}$	magnetic field vector
$\mathbf{J}$	current density
ε_0	dielectric permittivity of vacuum
$\mathbf{P}$	electric polarization
V_e	scalar electric potential

Mechanical Field

$\mathbf{u}$	mechanical displacement
$\mathbf{a}$	acceleration
$\mathbf{f}_V$	mechanical volume force
$\boldsymbol{\sigma}$	Cauchy stress vector (Voigt notation)
$\mathbf{S}$	linear/infinitesimal strain vector (Voigt notation)
$\mathcal{B}$	solid mechanics stiffness operator

Piezoelectricity

$[\mathbf{c}^E]$	tensor of linear mechanical elasticity moduli at constant electric field
$[\mathbf{e}]$	tensor of piezoelectric coupling coefficients

Function Spaces

$L^p(\Omega)$	Space of p -times Lebesgue integrable functions u
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$$\int_{\Omega} |u(x)|^p \, d\Omega < \infty, \quad \Omega \subset \mathbb{R}^d, \, p \in \mathbb{R}^+$$

over a d -dimensional domain (cf. [Ada75] p. 22).

$W^{m,p}(\Omega)$ Sobolev spaces. With the two integers $m \geq 0$ and $1 \leq p \leq \infty$ and equipped with their corresponding norms the spaces

$$W^{m,p}(\Omega) = \{u \in L^p(\Omega) : D^\alpha u \in L^p(\Omega) \text{ for } 0 \leq |\alpha| \leq m, \\ \text{where } D^\alpha u \text{ is the weak derivative}\}$$

are called Sobolev spaces (c.f. [Ada75] p. 44 ff., [Fle06] Sec. A.1).

$H^1(\Omega)$ Hilbert Sobolev spaces. The spaces of square integrable functions, whose first derivatives in a weak sense are also square integrable

$$H^1(\Omega) = W^{1,2}(\Omega),$$

are Hilbert spaces when equipped with their corresponding scalar product. The closure of the space of infinitely often differentiable continuous functions that vanish on the boundary $C_0^\infty(\Omega)$ in the spaces $H^1(\Omega)$ is defined as

$$H_0^1(\Omega) = \{u \in H^1(\Omega) : u = 0 \text{ on } \partial\Omega\}$$

For a definition of the fractional spaces $H^{1/2}(\Omega)$ and the dual spaces $H^{-1/2}(\Omega)$ refer to [Fle06] Sec. A.1.

Software

CFS++	Coupled Field System in C++. Inhouse FEM code c.f. [Kal10].
cplreader	Reader for CFD data which produces HDF5 files for CFS++.
HDF5	Hierarchical Data Format
CGNS	CFD General Notation System
XDMF	eXtensible Data Model and Format