

Counting Graphs

An Introduction with Specific Interest in Phylogeny

(Dietmar Cieslik, University of Greifswald)

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Dietmar Cieslik

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Anliegen der *Greifswalder Seminarberichte Biometrie und Medizinische Informatik* ist es, die Zusammenarbeit im Berührungsfeld von Medizin, Biowissenschaften, Mathematik und Informatik zu fördern.

In diesem Sinne steht die Mitarbeit an den Publikationen allen denen offen, die sich einer solchen Vermittlung zwischen den Wissenschaften verpflichtet fühlen, und die dazu einen Beitrag leisten wollen.

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