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This thesis is concerned with the model-based feedforward control design for systems governed by quasilinear parabolic partial differential equations in one- and two-dimensional spatial domains. The main focus is on the realization of setpoint transitions along predefined reference trajectories. The approach is based on the semi-discretization of the partial differential equation using suitable finite difference schemes. In the case of boundary control it is shown that the resulting system of ordinary differential equations is differentially flat for general nonlinearities. This system-theoretic property allows for the parametrization of the state and control input in terms of a so-called flat output and its time derivatives, which constitutes a systematic approach to the trajectory planning and feedforward control design for the considered transition problems. The particular appeal of this approach relies in its comparatively low computational demands especially for systems exhibiting significant nonlinear effects. The contributions of this thesis are two-fold. At first, it is shown by analytic considerations that the flatness-based parametrization of the semi-discretized approximation for vanishing discretization mesh widths approaches the flatness-based parametrization of the original infinite-dimensional system given in terms of an infinite series. Conditions for the convergence of this series are explicitly derived for some system classes. Second, tailored numerical methods are presented, which on the one hand provide an empirical test for the convergence of the parametrizations and which on the other hand extend the applicability of trajectory planning and feedforward control based on finite difference semi-discretization to a broader class of systems. Notably, the construction of admissible profiles is considered. Especially in two-dimensional spatial domains, where the reference trajectories have to be consistent with the boundary conditions, this a crucial task for the trajectory planning and feedforward control design. Both an optimization-based approach and an approach motivated by the regularization of inverse problems are presented and perform well also in cases where no analytical solutions are available. Furthermore, grid adaptation methods are introduced to make use of non-equidistant grids for state and control input parametrization, which can significantly reduce the necessary computational cost. Finally, both the problem of state reconstruction and tracking error stabilization are addressed. Integrating the flatness-based trajectory planning and feedforward control design, model-predictive tracking control and suitable state estimation within a two-degrees-of-freedom control scheme allows for the compensation of disturbances and modelling uncertainties as well as for the control of unstable PDEs.