GEORG MAIER

SMOOTH MINIMUM ARC PATHS

CONTOUR APPROXIMATION WITH SMOOTH ARC SPLINES

EINGEREICHT ZUR ERLANGUNG DES DOKTORGRADES DER NATURWISSENSCHAFTEN
AN DER FAKULTÄT FÜR INFORMATIK UND MATHEMATIK DER UNIVERSITÄT PASSAU

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Abstract

Let P be a simple polygon with interior I and two disjoint edges designated as the start and the destination. A smooth arc path is a sequence of circular arcs and line segments joined smoothly and staying inside the closure of I. We elucidate the construction of a *smooth minimum arc path*, i.e. a start-destination smooth arc path with the minimal possible number of segments. Although this problem is well-known, it hasn't been solved yet. We present a mathematical characterization of possible solutions that enables a constructive approach leading to an $O(n^2)$ algorithm, where n denotes the number of vertices. However, in many practical applications our algorithm is even sub-quadratic.
In fact, our approach is more general since we do not restrict ourselves to polygons but to a broader class of bounding curves, namely piecewise restricted analytic curves. We were able to show our constructive characterization of solutions for this class of curves as well.

Zusammenfassung

Sei P ein einfaches Polygon mit Innengebiet I und zwei als Start und Ziel ausgezeichneten, disjunkten Kanten. Unter einem *smooth arc path* verstehen wir eine Folge von Kreisbögen und Strecken-Segmenten, die glatt zusammengesetzt sind und im Abschluss von I verlaufen. Wir interessieren uns für die Konstruktion eines *smooth minimum arc path*, d.h. für einen glatten Start-Ziel Pfad mit minimaler Anzahl an Segmenten. Obwohl das Problem seit längerem bekannt ist, konnte es bisher nicht gelöst werden. Wir präsentieren eine mathematische Charakterisierung möglicher Lösungen, die ein konstruktives Verfahren ermöglicht. Dieses Verfahren lässt sich schließlich als Algorithmus mit quadratischer Laufzeit (abhängig von der Anzahl der Ecken n von P) implementieren. In vielen praktischen Tests konnten wir jedoch eine subquadratische Laufzeit feststellen.
Tatsächlich ist unser Ansatz sehr viel allgemeiner. Wir lassen nicht nur Polygone als begrenzende Kurve, sondern eine weitaus breitere Klasse von Kurventypen zu, nämlich stückweise analytisch fortsetzbare Kurven. Für diese Kurventypen gelang es ebenso unsere konstruktive Beschreibung der Lösungen zu beweisen.

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Passau, March 2010 Georg Maier

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