

Helical Cone-Beam Computed Tomography using the Differentiated Backprojection

**Kegelstrahl-Spiralcomputertomographie unter
Verwendung der Differenzierten Rückprojektion**

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Abstract

This thesis examines the use of the differentiated backprojection (DBP) with subsequent inverse Hilbert transform (HT), which is a theoretically-exact and stable (TES) reconstruction algorithm, for helical cone-beam computed tomography. Special attention in this examination is attributed to the question if the method is suitable to process redundant data, i.e., data which have been measured but which are not necessary to achieve a TES reconstruction. Helical cone-beam CT has been chosen as an application because it is the scanning technique most common in modern CT scanners and because it requires new image reconstruction algorithms to account for the ongoing growth of CT detectors in terms of detector rows.

The first part of the thesis introduces the DBP-HT method in two and three dimensions and gives all relevant proofs. It also contains a section of original work about different strategies of computing a finite inverse Hilbert transform using Söhrngen's formula on discrete data. The second part of the thesis comprises three chapters presenting original research: First, the general properties of the DBP-HT method are examined and different implementation strategies are compared. Subsequently, a new algorithm is introduced, which allows for the first time to perform a TES reconstruction while beneficially including all measured data at maximum pitch. Finally we present an extension of this algorithm which enables TES reconstruction using all data at arbitrary pitch. The thesis concludes with a discussion of the results and an outlook onto possible further work.

Zusammenfassung

Diese Arbeit untersucht die Verwendung der Differenzierten Rückprojektion (*differentiated backprojection*, DBP) mit anschließender inverser Hilberttransformation (HT) in der Kegelstrahl-Spiralcomputertomographie, wobei die DBP-HT ein mathematisch exakter und stabiler (*theoretically-exact and stable*, TES) Rekonstruktionsalgorithmus ist. Besondere Aufmerksamkeit wird hierbei der Frage gewidmet, ob die Methode zur Verarbeitung redundanter Daten geeignet ist, d.h., zur Verarbeitung von CT-Daten, die zwar gemessen wurden, zur Erzielung eines mathematisch exakten und stabilen Rekonstruktionsergebnisses aber nicht notwendig wären. Die Kegelstrahl-Spiralcomputertomographie wurde hierbei als Anwendung gewählt, da sie die am weitesten verbreitete Scantechnik moderner CT-Geräte darstellt und da sie neue Bildrekonstruktionsalgorithmen erfordert, um der stetigen Zunahme an Detektorzeilen in der CT Rechnung zu tragen.

Der erste Teil der Arbeit stellt die DBP-HT sowohl im zwei- als auch im dreidimensionalen Raum unter Erbringung der relevanten Beweise vor. Er enthält außerdem eine Darstellung über unterschiedliche Strategien zur Implementierung einer finiten inversen Hilberttransformation auf diskreten Daten unter Verwendung der Formel von Söhngen. Der zweite Teil der Arbeit umfasst anschließend eigene Untersuchungen: Zunächst werden die generellen Eigenschaften der DBP-HT untersucht und unterschiedliche Implementierungsstrategien verglichen. Anschließend wird ein neuer Algorithmus vorgestellt, der erstmalig eine mathematisch exakte und stabile Rekonstruktion unter Nutzung aller gemessenen Daten bei maximalem *pitch*-Wert ermöglicht. Zuletzt wird eine Erweiterung dieses Algorithmus erläutert, die eine mathematisch exakte und stabile Rekonstruktion unter Nutzung aller Daten bei beliebigem *pitch*-Wert erlaubt. Die Arbeit schließt mit einer Diskussion der Ergebnisse und einem Ausblick auf mögliche künftige Forschungsthemen.

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