

# **A Phase-Field Model for Technical Alloy Solidification**

Von der Fakultät für Georessourcen und Materialtechnik der  
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