

Berichte aus der Psychologie

Hermann Rodenhausen

**Semantic and Syntactic Levels
of Mathematical Thinking and Learning**

Theoretical Concepts
Empirical Results

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