

The Symbol of a Markov Semimartingale

D I S S E R T A T I O N

zur Erlangung des akademischen Grades

Doctor rerum naturalium

(Dr. rer. nat.)

vorgelegt

der Fakultät Mathematik und Naturwissenschaften
der Technischen Universität Dresden

von

Schnurr, Jan Alexander

geboren am 13.09.1979 in Dillenburg

Gutachter: Prof. Dr. René L. Schilling
 Prof. Dr. Volker Nollau
 Prof. Dr. Niels Jacob

Eingereicht am: 16.12.2008

Tag der Disputation: 27.04.2009

Berichte aus der Mathematik

Jan Alexander Schnurr

The Symbol of a Markov Semimartingale

Shaker Verlag
Aachen 2009

Bibliographic information published by the Deutsche Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Dresden, Techn. Univ., Diss., 2009

Copyright Shaker Verlag 2009

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 978-3-8322-8244-8

ISSN 0945-0882

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: www.shaker.de • e-mail: info@shaker.de

Contents

Introduction	3
Introduction and Overview	3
Notation	7
The General Theory of Stochastic Processes	9
1 Markov, Feller, Lévy	13
1.1 Continuous Negative Definite Functions	13
1.2 Markov Processes	15
1.3 Semigroups and Different Kinds of Generators	18
1.4 Feller Processes, Feller Semigroups and Their Generators	23
1.5 Lévy Processes	25
2 Semimartingales and Stochastic Integration	31
2.1 Finite Variation and Localization	31
2.2 Semimartingales and Their Characteristics	34
2.3 Stochastic Integration	40
2.4 Square Bracket and Martingale Preservation	44
2.5 Itô's Formula	47
2.6 Stochastic Differential Equations	48
3 Itô Processes and Feller Semimartingales	59
3.1 Feller Semimartingales	59
3.2 More on Itô Processes	69
4 The Symbol of a Stochastic Process	73
4.1 Definition and First Example	73
4.2 The Symbol of an Itô Process	73
4.3 Unbounded Symbols	77
4.4 Properties of the Symbol	81
5 The Symbol Associated to the Solution of an SDE	82
5.1 Lévy-driven SDE. The Symbol of the Solution	82
5.2 Itô-driven SDE. The Bivariate Symbol	85
5.3 The Symbol of a General SDE	87

6	Some Applications	89
6.1	Symbol, Generator and Characteristics	89
6.2	Indices	90
6.3	Moment Estimates for Feller processes	95
7	Processes Used in Mathematical Finance	97
7.1	COGARCH: Symbol and Generator	97
7.2	Ornstein-Uhlenbeck Process, Revisited	104
A	The Fine Topology	105
B	(Counter-)examples	106
C	Friedrichs mollifier	109
	Index of Notation	114