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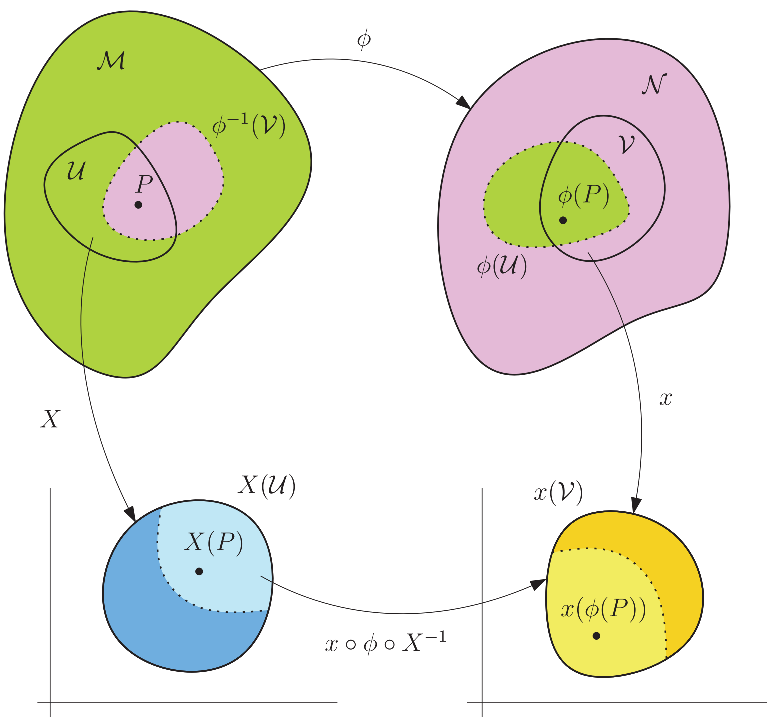
HEFT 44

Differential Geometry Applied to Continuum Mechanics



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Preface by the Editor

The astonishing pace in the development of the finite element method (FEM) in engineering sciences and its broad application in the industry and in engineering practice have entered the areas of soil mechanics and geotechnical engineering long ago. Although the perception of a soil continuum is a controversial issue, it is generally accepted that numerical simulations based on FEM can considerably improve the understanding of the physical processes involved in situ and the interpretation of measuring data of experimental tests.

Systematically, the notion of a continuum is part of the mathematic branch of differential geometry. Therefore, it is advantageous to analyze and to discuss the topics of continuum mechanics, in particular soil mechanics, by applying the geometric terminology. Accordingly, the soil continuum shall be understood as a differentiable manifold that does not need to have a Euclidian structure, and its stress and density states are be described by coordinate-independent tensor fields. Engineers not acquainted with tensor calculus and the geometric method are at a disadvantage, because they do not have full command of the scientific fundamentals of their discipline and thus can hardly benefit from new developments. They will moreover run the risk of blindly trusting the results of numerical simulations instead of questioning them.

The presented work is a fundamental introduction into differential geometry and its application to continuum mechanics. It is addressed at scientific engineers, but also at engineers in practice and graduate students interested in the field. Another objective of the work is to revise the theoretical fundamentals of the Arbitrary Lagrangian-Eulerian (ALE) formulation of continuum mechanics by placing emphasis on the geometric background. The ALE formulation can be seen as a unification of the Lagrangian and Eulerian formulations in order to combine the advantages of both viewpoints. It is currently a topic of research at the Soil Mechanics and Geotechnical Engineering Division, in which the penetration of piles into sand is being simulated numerically by using a finite element model. A publication in the institute series is being prepared.

The author currently is a research associate at the Technical University of Berlin. He was able to investigate the topic during his research activity, which is gratefully acknowledged here. Parts of the work on the ALE formulation have been carried out with the financial support of the DFG (German Research Foundation), which is also gratefully acknowledged.

Stavros A. Savidis
Berlin, February 2009

Preface by the Author

In the year 2004/2005, after my studies of civil engineering and becoming a research associate at the Soil Mechanics and Geotechnical Engineering Division, I attended a lecture by Prof. Dr.-Ing. Gerd Brunk at the Technical University of Berlin. The lecture was about tensor analysis and continuum physics, but it made me wonder since geometry was predominant, and ”index gymnastics” and mechanics were solely treated in applications. Inspired by this lecture and the famous book by Marsden and Hughes, I began my research work on an Arbitrary Lagrangian-Eulerian (ALE) approach to the finite element simulation of penetration processes in sand. Because the continuum mechanical background is massive and essentially based on the geometry of point spaces, I have written down this paper with the initial objective to compile important formulae and basic results. However, the reader will notice that the final version goes beyond to some extend.

I would like to thank Prof. Dr.-Ing. Stavros A. Savidis who gave me the opportunity to investigate as a geotechnical engineer such a theoretic topic, and my colleague Dr.-Ing. Frank Rackwitz for discussion and helpful suggestions. Last but not least I would like to thank the developers of the LaTeX program for enabling everyone to do beautiful typeset of complex mathematics.

Disclaimer

This paper is not intended to serve as a monograph for specialists about differential geometry and continuum mechanics. Many interesting topics have been omitted and many of the presented key facts and basic results are stated without proofs; they may be found in the standard textbooks, e.g. [1, 2, 3, 4, 5, 6, 7, 8]. Comprehensive treatises on linear geometry and linear algebra are, for example, [9] and [10].

Daniel Aubram
Berlin, November 2008

Abstract

Differential geometry provides the suitable background to present and discuss continuum mechanics with an integrative and mathematically precise terminology. By starting with a review of linear geometry in affine point spaces, the paper introduces modern differential geometry on manifolds including the following topics: topology, tensor algebra, bundles and tensor fields, exterior algebra, differential and integral calculi. The tools worked out are applied subsequently to basic topics of continuum mechanics. In particular, kinematics of a material body and balance of mass are formulated by applying the geometric terminology, the principles of objectivity and material frame indifference of constitutive equations are examined, and a clear distinction of the Lagrangian formulation from the Eulerian formulation is drawn. Moreover, the paper outlines a generalized Arbitrary Lagrangian-Eulerian (ALE) formulation of continuum mechanics on differentiable manifolds. As an essential part, the grid manifold introduced therein facilitates a consistent description of the relations between the material body, the ambient space and the arbitrary reference domain of the ALE formulation. Not least, the objective of the paper is to provide a compilation of important formulae and basic results —some of them with a full proof— frequently used by the community. If practical, point arguments and changes in points within equations will be clearly indicated, and component and direct (or absolute) tensor notation will be applied as needed, avoiding a single-track approach to the subject.

Keywords: differential geometry; continuum mechanics; large deformations; Arbitrary Lagrangian-Eulerian; manifold; tensor analysis

Zusammenfassung

Die Differentialgeometrie bietet den geeigneten Hintergrund, um die Kontinuumsmechanik mit einer einheitlichen und mathematisch präzisen Terminologie darzulegen und zu diskutieren. Ausgehend von einem Rückblick auf die lineare Geometrie in affinen Punkträumen führt die Arbeit in die moderne Differentialgeometrie auf Mannigfaltigkeiten unter Berücksichtigung der folgenden Themen ein: Topologie, Tensoralgebra, Bündel und Tensorfelder, Äußere Algebra sowie Differential- und Integralkalküle. Die erarbeiteten Werkzeuge werden anschließend auf grundlegende Themen der Kontinuumsmechanik angewendet. Insbesondere wird die Kinematik eines materiellen Körpers und die Massenbilanz vom geometrischen Standpunkt heraus formuliert, das Prinzip der Objektivität von Tensoren und von Materialgleichungen wird untersucht, und es wird der Unterschied zwischen der Lagrange'schen und der Euler'schen Formulierung auf klärende Weise dargestellt. Desweiteren skizziert die Arbeit eine verallgemeinerte Arbitrary Lagrangian-Eulerian (ALE) Formulierung der Kontinuumsmechanik auf differenzierbaren Mannigfaltigkeiten. Als wesentlicher Bestandteil ermöglicht dabei die eingeführte Gittermannigfaltigkeit eine konsistente Beschreibung der Beziehungen zwischen dem materiellen Körper, dem umgebenden Raum und dem beliebigen Referenzgebiet der ALE Formulierung. Nicht zuletzt besteht die Zielsetzung der Arbeit darin, wichtige Formeln und grundlegende Ergebnisse auf den behandelten Gebieten teilweise auch mit vollständigem Beweis zusammenzustellen. Sofern es zweckmäßig erscheint, werden Punktargumente und der Wechsel der Bezugspunkte in den Gleichungen hervorgehoben. Außerdem wird je nach Bedarf sowohl die Komponentenschreibweise, als auch die direkte oder absolute Schreibweise von Tensoren angewendet und dadurch ein eingleisiges Vorgehen vermieden.

Schlagworte: Differentialgeometrie; Kontinuumsmechanik; große Verformungen; Arbitrary Lagrangian-Eulerian; Mannigfaltigkeit; Tensoranalysis

God is a geometer.

—Plato

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