

Analog Codes for Analysis of Iterative Decoding and Impulse Noise Correction

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Abstract

We provide a very intuitive illustration of the convergence behavior of the sum-product algorithm for the iterative decoding of analog Turbo-like codes (or simply analog codes) by subdividing the parity-check matrix of an analog code into several *cycle-free* sub-parity check matrices. We show that the decoding algorithm can be intuitively illustrated as iteratively projections onto individual constituent code spaces defined by the sub-parity check matrices and end up in the least-squares solution. Formal proof is given for analog product codes. Based on the geometrical illustration, an improvement of the decoding speed can be expected if the constituent codes are designed to be close to orthogonal.

Analog Reed-Solomon (RS) codes as a special class of analog codes can be efficiently decoded by some well-known existing algorithms for impulse noise correction when the background noise is considerable low in amplitude. We designed a novel algorithm called Syndrome-Repairing Algorithm for impulse noise correction which turns out to be much more robust against background noise than all the existing algorithms and can achieve almost the least-squares solution as if there was no impulse noise present. Applications of Analog RS codes in practical OFDM (DMT) systems are also studied. By regarding the transmitter side as an analog RS codes, the impulse noise can be corrected without using additional standard error-control codes.

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