

Chair for Computation in Engineering
Fakultät für Bauingenieur- und Vermessungswesen
Technische Universität München

**High-order finite elements for
material and geometric nonlinear
finite strain problems**

Ulrich Heisserer

Vollständiger Abdruck der von der Fakultät für Bauingenieur- und Vermessungswesen der
Technischen Universität München zur Erlangung des akademischen Grades eines

Doktor-Ingenieurs

genehmigten Dissertation.

Vorsitzender: Univ.-Prof. Dr.-Ing. Dr.-Ing. habil. G. H. Müller

Prüfer der Dissertation:

1. Univ.-Prof. Dr.rer.nat. E. Rank
2. Priv.-Doz. Dr.-Ing. St. Hartmann,
Universität Kassel

Die Dissertation wurde am 24.09.2007 bei der Technischen Universität München eingereicht
und durch die Fakultät für Bauingenieur- und Vermessungswesen am 31.01.2008 angenommen.

Berichte aus der Bauinformatik

Ulrich Heisserer

**High-order finite elements for material and
geometric nonlinear finite strain problems**

Shaker Verlag
Aachen 2008

Bibliographic information published by the Deutsche Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: München, Techn. Univ., Diss., 2008

Copyright Shaker Verlag 2008

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 978-3-8322-7703-1

ISSN 1612-6262

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: www.shaker.de • e-mail: info@shaker.de

Zusammenfassung

Finite Elemente hoher Ordnung (p -Version) werden zur Simulation von geometrisch und materiell nichtlinearen Problemen angewandt. Neben hyperelastischen Materialien wird ein viskoplastisches Modell mit inneren Variablen verwendet. Das Algebro-Differentialgleichungssystem, welches aus der räumlichen Diskretisierung der schwachen Form entsteht, wird mit der Backward-Euler Methode zusammen mit dem Mehrebenen-Newton-Verfahren gelöst. Um den Prozess des kalt-isostatischen Pressens effizient abzubilden, werden ein axialsymmetrisches Element für große Dehnungen, Reaktionskräfte und Folgelasten für die p -Version abgeleitet. Analytische Vergleichslösungen zeigen, dass die p -Version volumetrisches Locking auch für große Dehnungen überwindet. Die effiziente Anwendung der entwickelten Methoden auf einaxiales und isostatisches Pressen von Metallpulvern wird demonstriert. Ein komplexes Validierungsbeispiel zeigt gute Übereinstimmung mit dem Experiment.

Abstract

For the simulation of geometric and material nonlinear problems implicit high-order (p -version) displacement-based finite elements are applied. Beside hyperelastic materials a finite strain viscoplasticity model with internal variables is considered. We apply the combination of Backward-Euler integration and Multilevel-Newton algorithm to solve the system of differential-algebraic equations resulting from the space-discretized weak form. For an efficient modeling of the cold isostatic pressing process an axisymmetric finite strain element, reaction forces and follower loads are derived in the p -version context. We demonstrate that the p -version can overcome volumetric locking also in the finite strain case. An adaptive time-stepping algorithm is presented to perform simulations of metal powder compaction. We report applications to die-compaction and isostatic pressing processes, and a complex validation example where a good agreement to experimental data is achieved.

Acknowledgements

I would like to thank all those people who contributed in any way to this dissertation. First of all, I thank my supervisor Prof. Ernst Rank for his continuous support and the opportunity to work in a very stimulating environment on the GIF project. PD Dr.-Ing. Alexander Düster was not only significantly involved in the definition of the project, but also had always an open door for discussions and dared to join the adventures in Israel.

I owe Dominik Scholz very much for introducing me to the world of high-order finite elements and supervising my diploma thesis. Hanne Cornils not only managed to ‘run’ our chair, she indeed was the heart and soul, I wish her all the best. Especially, I want to thank my officemate Petra Wenisch for good times and her friendship. Thanks go to all colleagues, with whom I share many pleasant memories and look back to some thrilling (table) soccer games.

For a friendly and fruitful cooperation, I would like to thank all colleagues of this GIF project, namely Prof. Moshe P. Driel, Prof. Nahum Frage, Prof. Stefan Hartmann, Prof. Stefan Holzer, and Prof. Zohar Yosibash as well as Wolfgang Bier, Idit Cohen, Magda Martins-Wagner, and Moti Szanto. I specially want to thank Zohar Yosibash and his group for the hearty hospitality during my stays in Israel.

Furthermore, I want to thank Stefan Hartmann for being the second reviewer of this thesis and his very constructive and detailed remarks. Additionally, I thank Prof. Gerhard Müller for chairing the examination.

With gratitude, I think of all those people who benevolently fostered my development — my family, teachers, and friends in Friedberg, Munich, and beyond. And Andrea, thanks for all the fish.

The support by the German-Israeli Foundation of Scientific Research and Development under grant I-700- 26.10/2001 is gratefully acknowledged.

Contents

Notation	v
1 Introduction	1
1.1 The process of cold isostatic pressing (CIP)	1
1.2 Outline of the thesis	3
2 Basic continuum mechanics	7
2.1 Kinematics	7
2.2 Balance principles and stress tensors	11
2.3 Constitutive models	15
2.3.1 Hyperelasticity	16
2.3.2 Powder plasticity	19
3 Variational formulation	25
3.1 Variational formulation	25
3.2 Linearization	28
3.3 Linearization of the variational equation	30
3.3.1 Directional derivative of internal virtual work	31
3.3.2 Follower load: directional derivative of external virtual work	32
4 Discretization	39
4.1 Spatial discretization by the finite element method	40
4.2 Hierarchical shape functions for high-order finite elements	43
4.2.1 The one-dimensional hierarchic basis	43
4.2.2 Hierarchic shape functions for quadrilaterals	46
4.2.3 Inter-element continuity	50
4.3 Mapping	50
4.3.1 Mapping concepts	52
4.3.2 Representation of rigid body modes	54
4.4 Error control	58
4.5 A note on temporal discretization	60
4.6 Discretized linearized variational form	61
4.6.1 Column matrix representation of the tensorial quantities	61
4.6.1.1 Strain tensor and related quantities	61
4.6.1.2 Stress tensor and related quantities	64
4.6.2 Spatial discretization of the linearized weak form	65

4.6.3	Element level quantities and their assembly	76
4.6.4	Numerical integration	78
4.6.5	DAE system and time discretization	82
4.7	Solution of the global system	84
4.7.1	Newton-Raphson method (NRM)	86
4.7.2	Multi-level Newton algorithm (MLNA)	86
4.7.3	Evaluation on element level	88
4.8	Elimination of interior degrees of freedom	91
4.9	Reaction Forces	95
4.9.1	Computing reaction forces	95
4.9.1.1	Motivation	96
4.9.1.2	Lagrange multiplier method for the DAE-system	96
4.9.1.3	Penalty function method	101
4.9.1.4	Reaction force in the context of the p -version	103
4.9.1.5	Verification of reaction forces	105
4.9.2	Application to the investigation of the St.Venant-Kirchhoff material	111
5	Application, adaption, and numerical studies	113
5.1	Finite strain axisymmetric p -version element	113
5.2	Follower load for the p -version: implementation and verification	115
5.2.1	Bending strip	115
5.2.2	Pressure on a thin circular plate	118
5.3	p -FEM is locking free for finite strain hyperelasticity	121
5.3.1	The locking problem and remedies	121
5.3.2	Thick-walled sphere under internal pressure	123
5.3.2.1	First investigation: p -extension	123
5.3.2.2	Second investigation: h -extension for fixed p	129
5.4	Simulating powder metallurgy	132
5.4.1	Die compaction examples	132
5.4.1.1	Cylinder	132
5.4.1.2	L-shaped domain	133
5.4.2	Cold isostatic pressing	139
5.4.3	Rubber isostatic pressing	146
5.4.3.1	Spherical cavity	147
5.4.3.2	Tube for vacuum circuit breakers	147
5.4.4	Experimental validation of a complex process	151
5.4.4.1	CIP of a powder cylinder with a rigid spherical inset	151
5.4.4.2	Modeling the process	151
6	Conclusion	157
A	List of publications originating from the GIF collaboration	161

B	List of symbols	163
B.1	Scalars	163
B.2	Shape functions and Ansatz spaces	164
B.3	Matrices and column matrices	165
B.4	Geometry: points, vectors, elements and mapping	166
B.5	Tensors	167
B.6	Work, potential energy & strain energy functions	169
B.7	Operators	169
	Bibliography	171