

**Linear Robust Control of a Nonlinear and Time-varying Process:
A Two-step Approach to the Multi-objective Synthesis of
Fixed-order Controllers**

Zur Erlangung des akademischen Grades eines

Dr.-Ing.

vom Fachbereich für Bio- und Chemieingenieurwesen
der Universität Dortmund
genehmigte Dissertation

von

Dipl.-Ing. Marten Völker
aus
Dortmund

Tag der mündlichen Prüfung: 16. Mai 2007

1. Gutachter: Prof. Dr. Sebastian Engell
2. Gutachter: Prof. Dr. Sigurd Skogestad

Dortmund, 2007

Schriftenreihe des Lehrstuhls für Anlagensteuerungstechnik
der Universität Dortmund (Prof.-Dr. Sebastian Engell)

Band 2/2007

Marten Völker

**Linear Robust Control of a Nonlinear
and Time-varying Process:**

A Two-step Approach to the Multi-objective Synthesis
of Fixed-order Controllers

D 290 (Diss. Universität Dortmund)

Shaker Verlag
Aachen 2007

Bibliographic information published by the Deutsche Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Dortmund, Univ., Diss., 2007

Copyright Shaker Verlag 2007

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 978-3-8322-6659-2

ISSN 0948-7018

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: www.shaker.de • e-mail: info@shaker.de

For Leonie

ACKNOWLEDGEMENT

First of all, I would like to thank Sebastian Engell, head of the Process Control Laboratory, for giving me the opportunity to dig deeper in the world of chemical engineering which, much to my astonishment five years ago, poses fascinating control problems, in particular due the complexity of physico-chemical first-principle models. I have got to know him as a challenging and universally informed supervisor with excellent generalisation skills who gave me the necessary freedom and the constructive advice to develop new ideas. I am impressed by his ability to quickly grasp, process, and discuss mathematical ideas which made me feel less alone in my specialised field of work. For carefully reading and co-assessing this work as a reputable expert in the field and for taking the long journey from Trondheim to Dortmund, I would like to thank Sigurd Skogestad. Thanks to Gabriele Sadowski, Andrzej Górak, and David Agar for being members of the examination committee. Special thanks to Gisela Hensche, who greatly contributed to my well-being during the doctoral examination, and to Dorothea Weber for the administrative help.

For accompanying many laborious, but nonetheless cheerful days and nights at the reactive distillation column and greatly contributing to the solution of many practical challenges, I am deeply indebted to Christian Sonntag, Christian Blichmann, Kai Dadhe, Gregor Fernholz, Joachim Richter, Herbert Broer, Siegfried Weiss, Katrin Kissing, Ralf Knierim, Werner Hoffmeister, Dirk Lieftucht, Tanet Wonghong, Achim Hoffmann, Carsten Buchaly, and Peter Kreis also played their part in solving many real-world problems related to the reactive distillation column. For the great team-work within the multimedia project Learn2Control, I would like to thank Andreas Liefeldt, Ernesto Elias-Nieland, Thomas Tometzki, Kai Dadhe, Manuel Remelhe, Christian Blichmann, Carsten Fritsch, Bastian Steinbach, Digpalsingh Raulji, and Christian Sonntag. For adding to the multi-objective controller synthesis software that emerged from this work and thereby introducing me to many helpful concepts of software design, I gratefully acknowledge the help of Ernesto Elias-Nieland. Thanks to Andreas Mayländer and Sven Birke for their computer support.

Many inspiring discussions with my colleagues of the Process Control Laboratory such as Kai Dadhe, Sabine Pegel, Andreas Liefeldt, Stefan Ochs, Olaf Stursberg, Guido Sand, Ralf Gesthuisen, Christian Sonntag, Tobias Neymann, Wolfgang Mauntz, Manuel Remelhe, Jochen Till, Sebastian Panek, Thomas Tometzki, Than-Ha Trinh, Karsten Klatt, Volker Rossmann, Ernesto Elias-Nieland, Maren Urselmann, Stefan Krämer, and Gregor Fernholz helped me to get a broader point of view of control- and optimisation-related research. For introducing me to the secrets of fluid separation processes, I appreciate the help of Joachim

Richter, Carsten Buchaly, and Markus Tylko. Likewise for thermodynamics, I received crucial advice from Michael Goernert, Matthias Kleiner, and Kai Kiesow. My former students, Claire Danquah, Kamol Limtanyakul, Christian Sonntag, Kai Kiesow, and Gaurang Shah, I thank for the good work and wish them all the best for their future careers.

Special thanks to Johan Löfberg for his expertise and helpfulness in ellipsoid approximation and the use of his essential tool Yalmip. He and Didier Henrion, who I thank for the nice and enlightening time at the LAAS in Toulouse, were permanently available and responsive with regard to my questions about semidefinite programming. I also profitted from discussions with Ralf Ebenbauer, Dimitri Peaucelle, Philipp Rostalski, Helfried Peyrl, Jochen Rieber, and Vinay Kariwala.

Last but not least: family and friends. I would like to express my deepest gratitude to my father, Ulrich Völker, for faithfully supporting my decision to become an engineer. For keeping me in touch and convincing the whole family of the engineering idea, I would like to thank my cousin Till Clausmeyer. Although I feel the urge to continue, I restrain a complete list of dear friends and relatives here, having good trust that all of you know how much I appreciate you for sticking around, cheering me up, and thus essentially contributing to the completion of this work.

ABSTRACT

Our approach to the development of linear control concepts for nonlinear uncertain processes emerged from the challenge to control a pilot-scale reactive distillation column for the semi-batch synthesis of methyl acetate. Methyl acetate synthesis by reactive distillation is well-known. Several linear control concepts have been suggested and have been tested with nonlinear simulation models. To date, however, publications where these control concepts have been tested in experiments on real plants are sparse, motivating the experiments conducted during this thesis. For the reactive distillation process, but for other processes as well, linear control is complicated by pronounced nonlinearities, slow time variations, considerable time delays, measurement as well as actuator uncertainties, and the fact that reliable control models are difficult to obtain.

To overcome these difficulties, our approach is characterised by the following central steps: first, a suitable control structure that enables an economic process operation is determined, employing a basic nonlinear process model that can be obtained relatively straightforwardly. Then, experiments are conducted to obtain a locally more accurate linear model and to compute a description of the mismatch between the linear model and the nonlinear process which are used to compute an optimal controller. A multi-objective synthesis procedure facilitates the formulation of all relevant control performance criteria, in particular robustness of the controller to the computed model uncertainties. To obtain a fixed-order controller that is practically applicable, the order of the optimal multi-objective high-order controller is reduced, paying particular attention to the conservation of robustness. The integrated approach is applied to the methyl acetate synthesis process, finally leading to a relatively simple multivariate PI controller so that the results can be transferred to industrial practice. The validity of the control concept is demonstrated in a series of control experiments that were performed at the real reactive distillation column.

Control-theoretic contributions of this thesis are the formulations of the multi-objective controller synthesis and the reduction step as convex linear-matrix-inequality (LMI) optimisation problems. As a central result, a new numeric optimisation method to efficiently solve these optimisation problems by a sequential solution of simpler quadratic programs is proposed. Convergence is proved and benchmark examples indicate significant improvements relative to current LMI solvers. To decrease the possibly limiting impact of the uncertainty description, another theoretic contribution deals with the computation of tight uncertainty bounds that are consistent with measured data. The formulation leads to a nonconvex optimisation problem that is solved by first computing a conservative solution by means of LMI techniques and then using the result as a starting point for a gradient-based local search.

KURZFASSUNG

Unser Ansatz zur Entwicklung von linearen Regelungen für nichtlineare und ungenau bekannte Prozesse entstand aus der Herausforderung eine semi-kontinuierliche Pilotreaktivrektifikationskolonne zur Synthese von Methylacetat zu regeln. Der Einsatz von Reaktivdestillation zur Methylacetatsynthese ist wohlbekannt: mehrere lineare Regelungskonzepte wurden vorgeschlagen und in rigorosen Modellen validiert. Bis heute gibt es jedoch kaum Veröffentlichungen, in denen diese Regelkonzepte in Experimenten belegt wurden. Aus diesem Grund bilden experimentelle Untersuchungen, sowohl zur Modellbildung als auch zur Validierung des Regelungskonzeptes, einen Schwerpunkt der vorliegenden Arbeit. Die lineare Regelung des Reaktivrektifikationsprozesses und anderer Prozesse im realen Anlagenbetrieb wird durch hervorgehobene Nichtlinearitäten, langsam zeitveränderliches Verhalten, beträchtliche Totzeiten, Ungenauigkeiten der Mess- und Stelleinrichtungen und durch die oft mangelnde Verfügbarkeit eines zuverlässigen Regelungsmodells erschwert.

Zur Bewältigung dieser realen Herausforderungen wird ein schrittweiser Ansatz verfolgt: zuerst wird, anhand eines einfachen, nichtlinearen Prozessmodells, eine geeignete Regelungsstruktur ausgewählt, die einen ökonomisch sinnvollen Betrieb ermöglicht. Um ein lokal genaueres dynamisches lineares Modell zu finden und um verbleibende Abweichungen des linearen Modells vom tatsächlichen nichtlinearen Prozess zu beschreiben, werden Experimente durchgeführt. Das lineare Modell sowie die Unsicherheitsbeschreibung werden zur Berechnung eines optimalen linearen Reglers eingesetzt. Die Formulierung aller praktisch relevanten Regelgütekriterien wird durch einen multikriteriellen Ansatz vereinfacht, der die Formulierung von Robustheitsnebenbedingungen zulässt. Um einen anwendbaren Regler vorgegebener Ordnung zu erhalten, wird der Optimalregler reduziert, wobei die Einhaltung der harten Regelgütekriterien berücksichtigt wird. Der integrierte Ansatz wird auf die Reaktivrektifikationskolonne angewendet und führt auf einen einfachen, industriell anwendbaren Mehrgrößen-PI-Regler, dessen Regelgüte in einer Reihe von Experimenten gezeigt wird.

Ein regelungstheoretischer Beitrag dieser Arbeit ist die Formulierung des multikriteriellen Syntheseproblems als konvexes LMI-Optimierungsproblem. Als zentrales Resultat wird ein neuer Ansatz vorgeschlagen, der das LMI-Optimierungsproblem durch eine Sequenz von numerisch einfacheren quadratischen Optimierungsproblemen löst. Die Konvergenz des Ansatzes wird bewiesen und numerische Experimente zeigen signifikante Vorteile im Vergleich zu aktuellen LMI-Optimierungsverfahren. Ein weiterer Beitrag handelt von der Berechnung von minimal konservativen Unsicherheitsschranken, die mit gemessenen Daten konsistent sind. Das resultierende nichtkonvexe Optimierungsproblem wird gelöst, indem zunächst eine konservative LMI-Optimierung durchgeführt wird, die einen Startpunkt für eine gradientenbasierte lokale Optimierung liefert.

CONTENTS

1. <i>Introduction</i>	1
1.1 Control of Integrated Reactive Distillation Processes	1
1.2 Case Study Example on Reactive Distillation	1
1.3 Previous Work	3
1.4 Contribution of this Thesis	4
1.5 Structure of this Thesis	6
2. <i>Case Study Example: Methyl Acetate Synthesis in a Pilot-scale Reactive Distillation Column</i>	7
2.1 Methyl Acetate Synthesis by Reactive Distillation	7
2.2 The Pilot Plant	8
2.3 Process Instrumentation and Control System	8
3. <i>Notation and Basic Concepts</i>	11
3.1 Vector and Matrix Notation	11
3.2 Frequently Used Operators of Linear Algebra	11
3.3 Signals	12
3.3.1 Signal Norms	13
3.4 Input-output Analysis of Dynamic Systems	14
3.5 Continuous-time Linear Systems	15
3.5.1 Stability, Norms, Hardy Spaces	16
3.6 Sampled-data Linear Systems	19
3.6.1 Stability, Norms, Hardy Spaces	19
3.7 Framework for the Analysis of Stochastic Processes	20
4. <i>Control Structure Selection</i>	21
4.1 Introduction	21
4.2 Control Structure Selection for the Pilot Plant	22
4.2.1 Preselection of Manipulated and Controlled Variables	24
4.2.2 Process Operability	24
4.2.3 Dynamic Process Operability	26
5. <i>Model Refinement & Model Uncertainty: A Data Based Approach</i>	29
5.1 Motivation & Overview	29
5.2 Variance & Bias	29

5.3	Linear System Identification for Control	31
5.3.1	Motivation	31
5.3.2	Preliminaries	32
5.3.3	Control-relevant Model Accuracy	33
5.3.4	Choice of Identification Signals, Experiment Design	34
5.3.5	Data Pretreatment	35
5.3.6	Regression	37
5.3.7	Model Reduction	40
5.3.8	Case Study on Reactive Distillation	41
5.4	Computation of Error Bounds for Robust Control	47
5.4.1	Basic Idea of Model Error Modelling	47
5.4.2	Review of Known Approaches	49
5.4.3	Model Error Modelling	51
5.4.4	Case Study on Reactive Distillation	56
6.	<i>Design of Linear Controllers: a 2-Step Procedure</i>	59
6.1	Introduction	59
6.2	Multi-objective Control	61
6.3	Generalised Plant	64
6.4	Multi-objective Control Performance Computation	66
6.4.1	Introduction	66
6.4.2	State-space Methods	66
6.4.3	Finite-gridding Approach	67
6.4.4	Frequency- and Time-Domain Discretisation of the Performance Criteria	68
6.4.5	Finite-dimensional Youla Parametrisation	69
6.4.6	Formulation of Optimisation Problem 6.4.1	71
6.4.7	A Novel Method for Solving a Quadratic Problem with LMI Constraints	74
6.4.8	Benchmarking the Controller Synthesis Method by Academic Examples	82
6.4.9	Case Study on Reactive Distillation	85
6.5	Controller Reduction	87
6.5.1	Introduction	87
6.5.2	Preliminaries	88
6.5.3	The Objective Function of the Frequency-weighted Approximation	88
6.5.4	Conservation of Nominal Stability	90
6.5.5	Conservation of $\mathcal{H}_\infty$ Norm Constraints	91
6.5.6	Conservation of Signal-to-peak Norm Constraints	92
6.5.7	Computation of Standard Controllers	92
6.5.8	Academic Examples	93
6.5.9	Case Study on Reactive Distillation	99

7. <i>Experimental Evaluation of the Control Concept for the Case Study</i>	101
7.1 Setpoint Change and Disturbance Rejection Scenario	101
7.2 Setpoint Change Scenario	103
7.3 Discussion	104
8. <i>Computation of Minimum Data-consistent Uncertainty Bounds</i>	107
8.1 Introduction	107
8.2 Literature Review	107
8.3 A Bandwidth Criterion for Tightness of Uncertainty Bounds	109
8.4 Setup	111
8.5 Problem Definition	111
8.6 Formulation of the Finite-dimensional Optimisation Problem	112
8.6.1 Finite-dimensional Series Expansion ($n_{\bar{u}} = 1$)	112
8.6.2 Formulation of the Objective Function ($n_{\bar{u}} = 1$)	113
8.6.3 Formulation of Data-consistency Constraints ($n_{\bar{u}} = 1$)	113
8.6.4 General case ($n_{\bar{u}} \geq 1$)	113
8.6.5 Final Optimisation Problem	114
8.7 Solution of the Nonconvex Optimisation Problem	114
8.7.1 First Step: Approximate Convex Solution	114
8.7.2 Second Step: Nonlinear Optimisation	116
8.8 Case Study on Reactive Distillation	117
8.8.1 Experimental Data & Linear Model	117
8.8.2 Computation of Error Bounds	118
8.9 Discussion	121
9. <i>Conclusions</i>	123
9.1 Outlook: Reactive Distillation	126
9.2 Outlook: Numerical Optimisation	126
9.3 Outlook: Control Structure Selection	126
<i>Appendix</i>	129
A. <i>Linear Algebra</i>	131
A.1 Vector and Matrix Norms	131
A.1.1 Vector p-norm	131
A.1.2 Induced Matrix Norms	131
A.1.3 Other Matrix Norms	132
A.2 Schur Complement	132
B. <i>Deterministic Signals and Systems</i>	133
B.1 Sampled-data Systems	133
B.1.1 SISO Case	133

B.1.2	MIMO Case	134
B.1.3	Transfer Matrix Representation	134
B.2	Linear Fractional Transformations	135
C.	<i>Stochastic Signals</i>	137
C.1	Ergodicity	137
C.2	Stationary Signals	137
C.2.1	Estimation of the Statistical Properties of Quasi-stationary Signals . .	138
C.2.2	The Power Spectral Density Function of a Quasi-Stationary Signal . .	138
D.	<i>Linear System Identification</i>	141
D.1	Choice of Identification Signals	141
D.1.1	Persistent Excitation and Identifiability	141
D.1.2	Crest Factor	141
D.2	The Prediction Error Framework	142
D.3	Orthonormal Basis Functions	142
D.3.1	Proof of (5.16a)	142
D.4	Numerical Values of the Orthonormal Basis Function Model	144
E.	<i>Multi-objective Control Performance Calculation</i>	145
E.1	Proof of Convergence of the New Algorithm	145
E.2	Implementation of the Algorithm	146
E.3	Minor Proofs	147
E.4	Numeric Values of the Synthesised Multi-objective Controllers for the Academic Examples	149
F.	<i>Controller Reduction</i>	151
F.1	Formulation as a Convex Optimisation Problem	151
F.1.1	Representation of the Objective Function as a Quadratic Problem (6.64b) 151	
F.2	Representation of the Performance Channels with the Reduced-order Controller 152	
F.2.1	Conservation of Nominal Stability	152
F.2.2	Conservation of Signal-to-peak Norm Constraints	153
F.3	Numeric Values of the Academic Examples	153
G.	<i>Minimum Uncertainty Bounds</i>	155
G.1	Numeric Details	155
G.1.1	Choice of Basis Functions for Strictly Positive Definite $\mathbf{P}_k$	155
G.1.2	Pruning of Constraints	155
G.1.3	Regularisation	156
G.2	Numeric Values for Case Study on Reactive Distillation	156

<i>Symbols, Units and Operators</i>	157
Symbols	157
Units	157
Operators	158
<i>Bibliography</i>	159
<i>Curriculum Vitae</i>	169

LIST OF FIGURES

1.1	Photograph of the semi-batch reactive distillation column.	2
1.2	Scheme of the semi-batch reactive distillation column.	3
4.1	Profile of the vapour phase composition over the column as obtained by simulation of the rigorous model at time steps $t_k \in \{10, 15, 20, 25, \mathbf{30}, 35, 40\} \cdot 1,000$ s (time progress indicated by arrows, the bold line refers to 30,000 s), the numbering of the y-axes corresponds to the temperature measuring points in Fig. 1.2.	25
4.2	Quasi-steady-state characteristic diagram in the $x_{\text{MeAc}}-x_{\text{H}_2\text{O}}$ -plane after 30,000 s of constant inputs (heat flow = 3.667 kW) with fixed feed. Additionally, the corresponding values of the temperature in the upper part of the separation section are shown.	26
4.3	Quasi-steady-state characteristic diagram in the $x_{\text{MeAc}}-x_{\text{H}_2\text{O}}$ -plane after 30,000 s of constant inputs (heat flow = 3.667 kW) with fixed reflux ratio. Additionally, the corresponding values of the temperature in the upper part of the separation section are shown.	27
4.4	Comparison of the simulation of the rigorous model (sim) with experimental data (data).	28
5.1	Linear system identification setup.	32
5.2	Control-relevant model accuracy.	33
5.3	Quasi-steady-state characteristic diagrams of the CVs (heat flow = 3.667 kW).	41
5.4	Identification input signals including spectra.	42
5.5	Frequency response of the high-pass filter W_f	43
5.6	Bode plots of the high-order and the low-order models (scaled according to (5.20)).	45
5.7	Identification data (data) and the corresponding simulated outputs (sim) of the reduced-order linear model.	46
5.8	Internal structure of Δ_a	48
5.9	Multiplicative model error model.	52
5.10	Internal structure of Δ	53
5.11	Output-multiplicative uncertainty as a result of the identification scheme.	57
6.1	Uncertain control loop.	62
6.2	Generalised plant setup for multi-objective controller synthesis.	64

6.3	Time effort for the optimisations - <i>sequence</i> of quadratic problems vs. LMI formulation (solver <i>lmiab</i>).	78
6.4	Accuracy of the constraints - <i>sequence</i> of quadratic problems (S) vs. LMI formulation (L) (solver <i>lmiab</i>).	79
6.5	Time effort for the optimisations - <i>sequence</i> of quadratic problems vs. LMI formulation (solver <i>sedumi</i>).	80
6.6	Accuracy of the constraints - <i>sequence</i> of quadratic problems (S) vs. LMI formulation (M) (solver <i>sedumi</i>).	81
6.7	Generalised plant for mixed design.	82
6.8	Generalised plant for the reactive distillation case study.	85
6.9	Generalised plant with reduced-order controller.	90
6.10	Robustness channel for stability of reduced-order controller.	91
6.11	Academic example 1: characteristic variables.	95
6.12	Academic example 1: u for step in r for a simulation step size of 10^{-6}	95
6.13	Academic example 2: $\mathbf{e}$ for steps in $\mathbf{r}$	96
6.14	Academic example 2: $\mathbf{u}$ for steps in $\mathbf{r}$	96
6.15	Academic example 2: $\bar{\sigma}(\mathbf{T}'_{z_\infty w_\infty}(j\omega))$	97
6.16	Academic example 2: Bode diagram of $\mathbf{T}_{z_2 w_2}(j\omega)$ for the different controllers.	97
6.17	Academic example 3: characteristic variables.	98
6.18	Academic example 3: u for step in r for a simulation step size of 10^{-6}	98
6.19	Closed-loop step responses with the optimal control performance controller (ocp) and the reduced-order PI controller (red), all variables are scaled according to (5.20).	99
6.20	Closed-loop step responses with the optimal control performance controller (ocp) and the reduced-order PI controller (red) in unscaled coordinates.	100
6.21	Singular values of the frequency response of the weighted $\mathcal{H}_\infty$ norm constraint performance channel with the optimal controller (ocp) and the reduced PI controller (red).	100
7.1	Experimental setpoint change and disturbance rejection scenario (disturbances).	101
7.2	Experimental setpoint change and disturbance rejection scenario (controlled and manipulated variables)	102
7.3	Experimental setpoint change scenario.	103
8.1	Idea of minimum uncertainty bounds from Hindi et al. (2002).	108
8.2	First zero crossing of $ W_u(e^{j\omega}) _{dB}$ from below.	109
8.3	Shaping $W_u(e^{j\omega})$ using a monotonically decreasing weighting function $W_l(\omega)$	110
8.4	Multiplicative model error model.	112
8.5	Idea of the two-step procedure. Ellipsoids: — ellipsoid constraints, — super-set ellipsoid, . . . quadratic objective. Solution points: $\square$ globally optimal, $\circ$ convex-hull optimal, $\bullet$ locally optimal.	116
8.6	Validation data set.	117

8.7	Frequency response of the monotonicity weighting functions W_l	119
8.8	Maximum singular values of the frequency response of the resulting $\mathbf{W}_u$ transfer matrices - induced 2 norm, see Tab. 8.1 for the indices of the weighting functions.	119
8.9	Frequency response of the resulting $\mathbf{W}_u$ transfer matrix for weight #9. . . .	120
8.10	Data-consistency constraint over time, model is invalidated below the dashed line.	121
9.1	Overview of the methodology.	124
G.1	Pruning of ellipsoids - the feasible set is the exterior of the union of ellipsoids, which is completely determined by the solid ellipsoids: the dashed ellipsoids can be pruned.	156

LIST OF TABLES

2.1	Sequence of boiling points for the MeAc synthesis.	8
5.1	Model error model input and output sequences for different uncertainty representations.	52
6.1	Solution times in seconds for <i>sequence lmilab</i>	78
6.2	Objective function evaluation at the solution point (<i>sequence lmilab</i>).	79
6.3	Solution times in seconds for <i>sequence sedumi</i>	80
6.4	Objective function evaluation at solution point (<i>sequence sedumi</i>).	81
6.5	Results of the computation.	84
6.6	Parameters of the performance computation for the case study.	86
6.7	Controller reduction for the academic examples: high-order and reduced-order controllers.	93
8.1	Different monotonicity weighting functions W_l	119
8.2	Result of the optimisation for weight #9: objective functions.	120
D.1	Continuous-time pole-zero-gain representation of the identified high-order model (scaled according to (5.20)). The values in brackets denote the multiplicity which is evaluated at a tolerance of 0.0001.	144
E.1	Parameters of the performance computation for the academic examples.	149
E.2	Optimal controllers: decision vectors.	149
E.3	Controllers synthesised with <i>hinfmix</i>	150
F.1	Reduced-order controllers.	153