

# **Spatial Diversity in MIMO Communication Systems with Distributed or Co-located Antennas**

## **Dissertation**

zur Erlangung des akademischen Grades  
Doktor der Ingenieurwissenschaften (Dr.-Ing.)  
der Technischen Fakultät  
der Christian-Albrechts-Universität zu Kiel

vorgelegt von

**Jan Mietzner**

Kiel 2007

Jan Mietzner, *Spatial Diversity in MIMO Communication Systems with Distributed  
or Co-located Antennas*

Tag der Einreichung: 02. Oktober 2006

Tag der Disputation: 13. Dezember 2006

Dekan: Prof. Dr. rer. nat. Manfred Schimmler

Berichterstatter: Prof. Dr.-Ing. Peter Adam Höher

Dr.-Ing. habil. Wolfgang H. Gerstacker

Prof. Dr. Lajos Hanzo

Digital Communications

**Jan Mietzner**

**Spatial Diversity in MIMO Communication Systems  
with Distributed or Co-located Antennas**

Shaker Verlag  
Aachen 2007

**Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Kiel, Univ., Diss., 2006

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Printed in Germany.

ISBN 978-3-8322-6130-6

ISSN 1860-7535

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: [www.shaker.de](http://www.shaker.de) • e-mail: [info@shaker.de](mailto:info@shaker.de)

# Preface

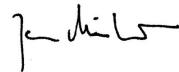
This thesis was developed during my work as a research and teaching assistant at the Information and Coding Theory Lab (ICT), Faculty of Engineering, Christian-Albrechts University of Kiel (CAU), Germany.

First of all, I would like to thank my advisor Prof. Dr. Peter Adam Höher for his enduring support and interest in my work. The working environment in Kiel was very enjoyable, which is largely his credit. In times of scarce financial resources he enabled me to visit numerous international conferences, which was a great experience.

Next, I would like to thank Dr. Wolfgang Gerstacker from the University of Erlangen-Nuremberg, Germany, and Prof. Dr. Lajos Hanzo from the University of Southampton, U.K., for evaluating this work. I am indeed very happy that Prof. Höher was able to win them for this task. The various interesting discussions, especially with Dr. Gerstacker, were very helpful. Also many thanks to Prof. Dr. Manfred Schimmler, the dean of the Faculty of Engineering, and to Prof. Dr. Ludger Klinkenbusch, for setting up a prompt date for the thesis defence.

I would like to thank my former colleagues of the ICT and the Institute for Circuit and System Theory for the pleasant time in Kiel and many technical as well as non-technical discussions. Moreover, I would like to thank Dr. Xiao-Ming Chen (Thomson, Hanover, Germany) and Prof. Dr. Badri-Höher (FHTW University of Applied Sciences, Vechta-Diepholz-Oldenburg, Germany) for providing parts of the software used for this work. Many thanks also to our former students Jan Eick, Malte Kautza, and Ravisankar Natarajan for their valuable contributions. Finally, many thanks to Justus Fricke (ICT), Dr. David Haley (Cohda Wireless, Kent Town, Australia), James Howarth (Macquarie University, Sydney, Australia), and my former office mate Prof. Dr. Ingmar Land (Aalborg University, Denmark) for proof-reading parts of the manuscript.

Last but not least, I would like to thank my wife Annika Mietzner, who has accompanied and encouraged me throughout my complete Ph.D. studies, and my parents Günter and Ursula Mietzner for their continuous support.



Vancouver, March 2007



# Abstract

THE USE OF multiple antennas in wireless communication systems has gained much attention during the last decade. It was shown that multiple-antenna systems, called multiple-input multiple-output (MIMO) systems, offer huge advantages over single-antenna systems, both with regard to capacity and error performance.

Typically, quite restrictive assumptions are made in the literature on MIMO systems concerning the spacing of the individual antenna elements. On the one hand, it is typically assumed that the antenna elements at the transmitter and the receiver are co-located, i.e., they belong to some sort of antenna array. On the other hand, it is often assumed that the antenna spacings are sufficiently large, so as to justify the assumption of independent fading on the individual transmission links. From numerous publications it is known that spatially correlated links caused by insufficient antenna spacings lead to a loss in capacity and error performance. In the first part of the thesis, it is shown that this is also the case when the individual transmit or receive antennas are spatially distributed on a large scale. Possible applications include simulcast networks, reach-back links for wireless sensors, as well as wireless networks with cooperating relays. Specifically, it is proven that any spatially correlated MIMO system can be transformed into an equivalent (with regard to the resulting capacity distribution) spatially distributed MIMO system, and vice versa. Moreover, the asymptotic equivalence with regard to the pairwise error probability of space-time codes is proven. Correspondingly, MIMO systems with distributed antennas and MIMO systems with co-located antennas can be treated in a single, unifying framework.

This fact is utilized in the second part of the thesis, where appropriate transmit power allocation strategies are developed for MIMO systems with distributed or co-located transmit antennas. In particular, fading scenarios are taken into account that occur especially in distributed MIMO systems. Focus is on power allocation schemes that require solely statistical channel knowledge at the transmitter side, which can easily be acquired in practical systems. By means of analytical results, it is shown that significant performance gains in comparison to equal power allocation are achieved.

The third part of the thesis focuses on two problems that are of particular interest for MIMO systems with distributed transmit antennas. First, due to the distributed nature of the system, independent local oscillators are employed for up-converting the individual transmitted signals. This causes frequency offsets between the transmission links, which results in time-varying channel impulse responses. The impact of frequency offsets on the performance of different space-time coding techniques is analyzed, and possible counter measures are considered. Second, if the transmit antennas are spaced very far apart and no

timing advance techniques are employed, significantly different propagation delays occur that lead to intersymbol interference effects. To this end, suitable space-time coding and equalization techniques are identified, so as to maintain a diversity advantage in comparison to a single-antenna system.

**Keywords:** Wireless communications, MIMO systems, space-time codes, spatial fading correlation, distributed antennas, performance analysis, transmit power allocation, equalization, frequency offset. ★

# Kurzfassung

**M**OBILFUNKSYSTEME mit mehreren Sende- und Empfangsantennen, sog. Multiple-Input Multiple-Output- (MIMO-) Systeme, haben in den letzten zehn Jahren großes Interesse geweckt. Wie vielfach gezeigt wurde, bieten MIMO-Systeme hinsichtlich höherer Datenraten und geringerer Fehlerraten beachtliche Vorteile gegenüber Mobilfunksystemen mit nur einer Sende- und Empfangsantenne.

Typischerweise werden in der Literatur über MIMO-Systeme relativ strenge Annahmen bezüglich der Abstände der einzelnen Antennenelemente getroffen: Auf der einen Seite nimmt man normalerweise an, dass die Sende- und Empfangsantennen Teil eines Antennen-Arrays sind ("co-located antennas"). Auf der anderen Seite wird häufig angenommen, dass die Antennenabstände hinreichend groß sind, so dass man von statistisch unabhängigen Fadingprozessen auf den einzelnen Übertragungslinks ausgehen kann. Aus zahlreichen Publikationen ist bekannt, dass räumliche Korrelationseffekte – verursacht durch unzureichende Antennenabstände – zu Verlusten hinsichtlich der erreichbaren Daten- und Fehlerraten führen. Im ersten Teil der Arbeit wird gezeigt, dass dies ebenso der Fall ist, wenn die einzelnen Sende- oder Empfangsantennen räumlich verteilt sind (auf einer großen Skala). Mögliche Anwendungen sind zum Beispiel sog. Gleichwellennetze für Rundfunkanwendungen, drahtlose Sensornetze sowie Mobilfunknetze mit kooperierenden Zwischenstationen. Insbesondere wird gezeigt, dass (hinsichtlich verschiedener Performance-Kriterien) jedes räumlich korrelierte MIMO-System in ein äquivalentes räumlich verteilt MIMO-System überführt werden kann und umgekehrt. Demzufolge können MIMO-Systeme mit verteilten Antennen und MIMO-Systeme mit korrelierten Antennen in einem gemeinsamen theoretischen Rahmen behandelt werden.

Diese Tatsache wird im zweiten Teil der Arbeit ausgenutzt, in dem geeignete Strategien zur Verteilung der Sendeleistung auf die einzelnen (korrelierten oder verteilten) Sendeantennen entwickelt werden. Insbesondere wird auf Fading-Szenarien eingegangen, die speziell in verteilten MIMO-Systemen auftreten können. Die betrachteten Techniken benötigen ausschließlich statistische Kanalkenntnis auf der Sendeseite, welche in praktischen Systemen leicht zur Verfügung gestellt werden kann. Mit Hilfe analytischer Ergebnisse wird gezeigt, dass durch eine geeignete Verteilung der Sendeleistung deutliche Gewinne erzielt werden können.

Der dritte Teil der Arbeit befasst sich schließlich mit zwei Problemen, die insbesondere für MIMO-Systeme mit verteilten Sendeantennen von Interesse sind. Zum einen werden aufgrund der räumlichen Trennung der Sendeantennen unabhängige Frequenzoszillatoren zur Aufwärtsmischung der zu übertragenden Signale verwendet. Dies führt zu Frequenzversätzen und somit zu zeitvarianten Kanalimpulsantworten. Der Einfluss

solcher Effekte auf die Leistungsfähigkeit verschiedener MIMO-Übertragungstechniken wird analysiert, und mögliche Gegenmaßnahmen werden vorgeschlagen. Zum anderen, wenn die Sendeantennen räumlich sehr weit getrennt sind und keinerlei Techniken zur Signallaufzeitkompensation verwendet werden, treten deutliche Unterschiede zwischen den einzelnen Ausbreitungsverzögerungen auf, welche zu Intersymbol-Interferenz-Effekten führen. Dementsprechend werden geeignete Sender- und Empfängertechniken identifiziert, die einen Diversitätsgewinn im Vergleich zu einem Einantennensystem aufrecht erhalten.

**Stichwörter:** Mobilfunkkommunikation, MIMO-Systeme, Space-Time-Codes, räumliche Korrelation, verteilte Antennen, Performance-Analyse, Sendeleistungsverteilung, Entzerrung, Frequenzversatz.



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