

Classical Mechanics

Peter Ryder

Berichte aus der Physik

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Preface

Physics is the most basic of all natural sciences, and provides the foundation all the other natural and engineering sciences. Physicists concern themselves with questions such as:

- What are the fundamental objects which make up the universe?
- How do these objects interact with one another?
- What are the basic laws governing these interactions?

An introductory university physics course is traditionally divided into a number of subfields. A typical “table of contents” for such a course might include the following topics¹:

- Mechanics: kinetics (description of motion), dynamics (mass, force, acceleration), work and energy, momentum and angular momentum, frames of reference, harmonic oscillations, mechanical properties of matter, waves.
- Optics: the nature of light, reflection and refraction, lenses and mirrors, optical instruments, light as electromagnetic radiation, interference and diffraction, polarized light.
- Thermodynamics: heat and temperature, thermal properties, statistical mechanics and kinetic theory, states of matter and phase equilibria, the laws of thermodynamics, engines and heat pumps, entropy, thermodynamic potentials, transport processes,
- Electrodynamics: electrostatics (charges, electric fields), electric currents (Ohm’s Law), moving charges and magnetic fields (induction), magnetic properties of materials, time-dependent magnetic fields (induction), Maxwell’s equations and electromagnetic radiation.
- Special relativity theory: the constancy of the speed of light, the Lorentz transformation, relativistic kinetics, relativistic dynamics, the equivalence of mass and energy, relativistic transformations of physical quantities.

¹This is of course only an example; many variations are possible.

- Quantum physics: historical origins, the Schrödinger equation, formal basis of quantum physics, the hydrogen atom, atoms with many electrons, chemical bonds, molecules, basic nuclear physics.
- Statistical physics: quantum and “classical” statistics, Bose-Einstein, Fermi-Dirac and Boltzmann statistics, applications, statistical basis of macroscopic properties (work, heat, entropy).

The number of research fields in physics is of course basically unlimited. The following list is taken from the top level of the *Physics and Astronomy Classification Scheme* (PACS) of the *American Institute of Physics* (AIP), which is widely used for the classification of physics publications: elementary particles and fields, nuclear physics, atomic and molecular physics, electromagnetism, optics, acoustics, heat transfer, classical mechanics, fluid dynamics, physics of gases, plasma physics, physics of electric discharges, condensed matter physics, geophysics, astrophysics. Of these fields, some, such as particle physics and astrophysics, are very basic, i.e. located at the frontiers of our present knowledge of physics, whilst others, such as optics or fluid dynamics, belong to the broad category of applied physics.

A further distinction is made between *theoretical* and *experimental* physics (and physicists). The experimentalist designs experiments and makes measurements, and the theoretical physicist develops the mathematical formulation of the laws. The process is iterative: the theoretical physicist creates or modifies his theories on the basis of the experimental results, and the experimental physicist develops measurement strategies and analyzes the results in the light of the existing theories.

This book is an introduction to *classical* mechanics — classical in the sense of *non-relativistic*, i.e. limited to velocities which are small compared to the velocity of light. Most introductory university physics courses begin with classical mechanics. There are several good reasons for keeping to this tradition. Firstly, mechanics is one of the oldest branches of physics, having been developed by Newton in the 17th century, based mainly on the work of Galileo and Kepler². Secondly, mechanics provides good examples of the application of differential and integral calculus, which are usually taught also in the first undergraduate year. Thirdly, many basic physical concepts which are introduced in the mechanics course (mass, force, energy, momentum, frames of reference etc.) are required in most other branches of physics. Finally, mechanics presents fewer conceptual

²Newton himself wrote: “If I have seen farther than others, it is because I stood on the shoulders of giants.”

difficulties to the student than e.g. special relativity or quantum physics.

The book is based on lecture notes (originally in German) which accompanied a two-year introductory physics course at the University of Bremen, Germany (classical mechanics took up the first semester). Like the lecture notes, this book has been produced in two versions, one (the one you are reading now) suitable for printing and the other designed for viewing on the computer screen. The lecture notes were developed in a period in which the Internet was becoming increasingly important as a source of serious scientific information and teaching materials. Consequently, numerous internet sites are quoted — in the electronic version linked directly to the (PDF) document — at the appropriate places. In this book, the links were checked and, as required, updated immediately before publication.

Although this book is mainly aimed at those who are studying for a physics degree, it is also suitable for students of other natural sciences, mathematics or engineering who have physics as a subsidiary subject.

What makes planets go around the sun? At the time of Kepler some people answered this problem by saying that there were angels behind them beating their wings and pushing the planets around an orbit. As you will see, the answer is not very far from the truth. The only difference is that the angels sit in a different direction and their wings push inward.

RICHARD FEYNMAN

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