From Algebraic to Transcendental Numbers

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Preface

The definition of transcendental numbers as non-algebraic numbers divides the real number algorithms into two parts which intersection is the empty set. While the set of algebraic numbers is closed with the rational operations, a sum or a product of transcendental numbers can become algebraic.

Famous transcendental numbers are f.e. e, π, and the general logarithmic or trigonometric functions, which can also have algebraic results.

In this book, after an introduction, basic theorems of algebraic numbers, algebraic field extensions and quadratic number fields, the way from algebraic to transcendental numbers, basic theorems of transcendental numbers, transcendental field extensions, the numbers e and π, applications, the classification of real numbers, unsolved problems, and two old proofs of the transcendence of e are discussed. In the last chapter, fundamental theorems of number theory and mathematics are added.

This contribution is part of a sequence of studies in number theory. As in former volumes, the author has tried to find and cite the relevant literature so far as possible. The proofs are made or modified by the author; if literature was used, a notice is given. If there is no such hint, there is certainly no guarantee that similar proofs haven't been published by other authors before.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in January 2007 Uwe Kraeft

Choice of symbols

$\Rightarrow, \Leftarrow, \Leftrightarrow$	by this follows (in the given directions)
$\in$	is element of (is contained in)
$\cup$	union
$A=\{a,b,c\}$	an example of a set A with elements a, b, and c
N	set of natural numbers 1, 2, 3, ...
P	primes of N 2, 3, 5, ... (P^1 with the unity)
N^0	$N\cup\{0\}$
N^-	$\{-n;\ n\in N\}$, set of negative integers -1, -2, -3, ...
Z	$=N\cup\{N^-\}\cup\{0\}$, set of integers
Q	set of rational numbers a/b with $a\in Z$, $b\in N$
R	set of real number algorithms
Q(R)	Q or R
$Q(\sqrt{2})$	field extension $Q\cup\sqrt{2}$
L/K	field extension of K f.e. $Q(\sqrt{2})/Q$
C	=Q(i) or R(i) set of complex numbers a+ib
[a,b]	all x in the closed interval $a\leq x\leq b$
$=$	equal (identical) by axioms or definitions
$\cong$	so near as you want but not identical
$\approx$	about, rounded, can f.e. be approximated for great n
$\sim$	similar order of magnitude
$\equiv$	$a\equiv b \pmod c \Leftrightarrow a\equiv b_c \Leftrightarrow (a-b)/c\in Z$ for $a,b\in Z$, $c\in N$
$p,(q>1)\in N$	p,q are natural numbers and q>1
ln x	natural logarithm with $x=e^{\ln x}$
PT	Pythagorean Triple
FLT	Fermat's Last Theorem
f.e.	for example (e.g.)

Capital letters are used for names of persons and very famous theorems, formulae, or numbers.

The order of this sequence of texts on number theory is twofold. The order following the date of printing is given at the end of this book. Another grouping is got by the colours of the covers after disciplines as follows:

arithmetic number theory:	light blue
sequences and series:	dark green
Diophantine Equations:	orange
algebraic number theory:	dark red
topological number theory:	purple
analytic number theory:	dark blue
statistical number theory:	light green
special numbers:	dark yellow
textbooks:	light yellow

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