

# Non-matching Triangulations of Curvilinear Interfaces Applied to Electro-Mechanics and Elasto-Acoustics

Von der Fakultät Mathematik und Physik der Universität Stuttgart  
zur Erlangung der Würde eines Doktors der  
Naturwissenschaften (Dr. rer. nat.) genehmigte Abhandlung

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