

Evolution of Cooperation

The Prisoner's Dilemma and its Applications as
an Example

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*The chess-board is the world;
the pieces are the phenomena of the universe;
the rules of the game are what we call the laws of Nature.*

*The player on the other side is hidden from us.
We know that his play is always fair, just, and patient.
But also we know, to our cost, that he never overlooks a mistake,
or makes the smallest allowance for ignorance.*

T. H. HUXLEY 1825–1895,
Lay Sermons: A *Liberal Education*

Contents

Introduction	1
Overview	3
1 The Prisoner's Dilemma	5
1.1 Game Theoretical Formulation	5
1.1.1 Trivial Solutions	6
1.2 The iterated Prisoner's Dilemma	7
1.2.1 Stability	8
1.2.2 Simultaneous versus Alternating Decisions	9
1.3 Memory One Strategies	10
1.3.1 Prominent Representatives	10
I Increasing Memory and Number of Players	13
2 Introduction	15
2.1 Increasing the Memory Length	15
2.2 Increasing the Number of Players	16
3 Simultaneous Prisoner's Dilemma	17
3.1 The Game	18
3.1.1 Evolutionary Game Dynamics	20
3.2 Results	21
3.2.1 N players, N steps memory	21
3.2.2 2 players, M steps memory	23
3.3 Conclusions	25
4 Alternating Prisoner's Dilemma	29
4.1 The Game	30
4.2 Results	34
4.2.1 2 players, M steps memory	35
4.2.2 N players, N steps memory	37
4.3 Conclusions	41

A	Markov Chains	45
A.1	Stochastic Matrices	45
A.2	Application to the Prisoner's Dilemma	46
II	Adaptive Strategies	49
5	Introduction	51
5.1	Personal Experiences	51
5.2	Towards a New Strategy	52
6	Adaptive Strategies	55
6.1	Concept & Model	56
6.2	Evolutionary Simulations	57
6.3	Results & Conclusions	59
B	Parameter Optimization of <i>Adaptor</i>	61
B.1	Rigorous Approach	62
B.2	Evolutionary Optimization	65
C	Solving the Master Equations	71
C.1	Memory One Strategies	71
C.2	<i>Adaptor</i> against its own kin	74
C.2.1	Simultaneous <i>IPD</i>	75
C.2.2	Alternating <i>IPD</i>	81
C.3	<i>Adaptor</i> against Memory One Strategies	88
C.3.1	Simultaneous <i>IPD</i>	88
C.3.2	Alternating <i>IPD</i>	94
7	Summary & Outlook	99
	Bibliography	103
	List of Figures	109
	List of Tables	111
	Acknowledgements	113
	Curriculum Vitae	115