Complex and Other Structural Numbers in Number Theory and Sciences

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Preface

The first main step in mathematics and number theory was the abstraction of elements and counting. By the second step the rational numbers were invented for elements which can be parted so often as you want. The third idea was to realize that there are irrational numbers. A fourth step was the introduction of the zero and negative numbers. With the following fifth advance came the introduction of axioms; now you could define numbers by these or look for numbers which are following all or a part of the axioms. The last generalization was got by the sixth step and the introduction of structural numbers. These are all numbers which consist basically of more than one of the before deduced numbers or also of structural numbers, as f.e. complex and hypercomplex numbers, vectors, matrices, tensors, and others. All further generalizations lead to algebra.

The main subject of this text are the complex numbers in infinitesimal calculus and analytic number theory. The theory of functions is only used in its relevant parts. The more algebraic characteristics of complex numbers are already treated in former books of this series. The book ends with other structural numbers and applications of these and of complex numbers in mathematics and sciences.

This text is part of a sequence of studies in number theory. As in former contributions, the author has tried to find and cite the relevant literature so far as possible. The proofs are made or modified by the author; if literature was used, a notice is given. If there is no such hint, there is certainly no guarantee that similar proofs haven't been published by other authors before.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in July 2006 Uwe Kraeft

Choice of symbols

$\Rightarrow, \Leftarrow, \Leftrightarrow$	by this follows (in the given directions, see next side)
$\in$	is element of (is contained in)
$\cup$	union
$A=\{a,b,c\}$	an example of a set A with elements a, b, and c
N	set of natural numbers 1, 2, 3, ...
P	primes of N 2, 3, 5, ... (P^1 with the unity)
N^0	$N\cup\{0\}$
N^-	$\{-n; n\in N\}$, set of negative integers -1, -2, -3, ...
Z	$=N\cup\{N^-\}\cup\{0\}$, set of integers
Q	set of rational numbers a/b with $a\in Z$, $b\in N$
R	set of real number algorithms; Q(R) Q or R
$\underline{r}$	vector
$a=\alpha+i\beta$	$=\alpha+\beta i$ complex number with $\alpha,\beta\in Q(R)$ and $i^2=-1$
$a^*=\bar{a}=\alpha-i\beta$	conjugate complex number
$a=\alpha+i\beta$	$=r(\cos\varphi + i*\sin\varphi)$ in polar coordinates
	$r=\|a\|$ modulus or absolute value of a or length of vector $\underline{r}$,
	φ is called argument arg(a) or arg a or the phase angle
$\Re(a),\Im(a)$	real, imaginary part of a
C	=Q(i) or R(i) set of complex numbers
C^G	Gaussian integers with $(\alpha,\beta\neq 0)\in Z$
ω	root of unity (see chapter 2)
$w=\dfrac{az+b}{cz+d}$	linear transformation (see chapter 2)
(z_1,z_2,z_3,z_4)	cross ratio (see chapter 2)
$=$	equal (identical) by axioms or definitions
$\cong$	so near as you want but not identical
$\approx$	about, rounded, can f.e. be approximated for great n
$\equiv$	$a\equiv b \pmod c \Leftrightarrow a\equiv b_c \Leftrightarrow (a-b)/c\in Z, C^G$ $(\underset{C}{\equiv})$ for $a,b\in Z, C^G$, $(c\neq 0)\in N, C^G$

$$f'(z) = \frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\int_a^b f(t)dt = \int_a^b u(t)dt + i\int_a^b v(t)dt$$

$$\int_\gamma f(z)dz = \int_a^b f(z(t))z'(t)dt$$

(a_n)	sequence
ln x	natural logarithm with $x=e^{\ln x}$
PT	Pythagorean Triple
f.e.	for example (e.g.)

Capital letters are used for names of persons and very famous theorems, formulae, or numbers.

"If A is true, then and only then B is true" shall be identical with "A ⇔ B". This is symbolized and explained as follows:

$$A \underset{\text{"only then"}}{\overset{\text{"then"}}{\rightleftarrows}} B$$

That means in words, there exists a way from A to B, and B cannot be true if A isn't true, or if B is true, then A must be true. The symbol "A ⇔ B" shows the paths of logic in shortest and best way. It is f.e.

7 is only divided by 7 and 1 $\rightleftarrows$ 7 is a prime $\rightleftarrows$ f_6 has no zeros.

Therefore, "if A is true, then and only then B is true" or "A ⇔ B" doesn't mean that there exists no other way to and from B, what could be supposed from the statement alone. In any case, if A is true, B must be true, and if B is true, A must be true. Any other statement C ⇔ B mustn't be a contradiction and it must be in transitive way A ⇔ C.

A similar statement is "if and only if A is true, then B is true"; if A is true, then follows that B is true, and B can only be true if A is true, whence follows as before A ⇔ B.

In the statements above, it can be deduced from "B is false" that also A must be false, because, if it was true, also B would be true.

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