

# **A Review of Buckling in Oil Wells**

Mesfin Belayneh  
University of Stavanger  
2006





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## Foreword

The field of petroleum engineering has changed radically the past decade. The easy oil is produced, and new discoveries are often found at challenging locations. Examples are deeper waters, high pressure and high temperature regimes and at offshore locations. This put higher demands on the design and the execution of the drilling operations.

Pipe buckling is an issue that has many aspects. For extended reach and horizontal wells the string is sometimes put in compression. Too high axial force leads to stuck pipe and casing. In particular coiled pipe tubing operations suffers from buckling constraints. There is no doubt that the buckling of oil well tubulars is one of the key challenges for future advanced wells.

A number of publications are written on pipe buckling. However, readers may become confused because the models often look similar, but with a different scaling factor. Which solution is correct and which is wrong? The answer is that they are all correct based on their assumptions. Furthermore, most of the published literature assumes a frictionless environment, and, they assume perfectly straight and symmetric tubulars. This indicates that there is still work required to arrive at buckling models applicable for real wells.

Dr. Mesfin Belayneh analyzed published buckling literature to sort out these issues. The results are presented in this book. He has systemized the various approaches in a clear and concise way. This makes this book even more important; it can be used to build new and more realistic models in the future. Furthermore, the book contains new models never published before. I foresee that new or revised buckling solutions will be published in the years to come, with this book as an important reference.

I will congratulate Dr. Belayneh for his work to analyze oil well buckling. This book is a must for everyone challenging theoretical buckling issues.

Stavanger, February 2006  
Bernt S. Aadnoy, PhD  
Professor of Petroleum Engineering  
University of Stavanger

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## List of symbols

$A$	cross sectional area of drill pipe, $in^2, m^2$
$D_i$	diameter of pipe, $in, m$
$D_o$	diameter of well bore, $in, m$
$d_B$	diameter of bending structure, $in, m$ Eq. 4.56
$E$	Young's Modulus, $psi, kN/m^2, MPa$
$EI$	bending stiffness, $psi, kN.m^2$
$F_{hel,t}$	top helical buckling load, $lb, N$
$F_a$	applied load at the top of the tube, $lbf, N$
$F_E$	effective force, $lbf, N$
$F_R$	real/true force, $lbf, N$
$F_{cr}$	compression force for sinusoidal buckling, $lbf, N$
$F_{Hel}$	compression force of helical buckling, $lbf, N$
$g$	acceleration due to gravity, $fts^{-2}, ms^{-2}$
$G$	modulus of rigidity, $psi, MPa$
$I$	moment of inertia, $in^4, m^4$
$J$	polar moment of inertia, $in^4, m^4$
$k$	maximum permissible dogleg severity, $degree/100ft$
$L_{eff}$	effective length, $ft, m$
$L$	length of drill string, $ft, m$
$M$	total bending moment for the full contact solution Eq 3.48
$p$	pitch length, $ft, m$
$p_i$	pipe pressurized internally, $psi, MPa$
$p_o$	pipe pressurized externally, $psi, MPa$
$\bar{q}$	Distributed load, $lbf.ft^{-1}, Nm^{-1}$
$r$	radial clearance, $in, m$
$r$	radius of gyration, $in, m$ Eq.1.13
$R_3$	axial compressive load on the pipe, $lbf, N$
$R_l$	contact force, $lbf, N$
$R$	curvature of bending, $ft, m$
$s$	arc length, $ft, m$
$T$	torque, $ft-lbf, Nm$
$T$	axial tension, $lbf, N$
$T$	temperature field, $^{\circ}K$
$Tn$	tension load below the dogleg, $lbf, N$

$\Delta U$	change in stored energy, <i>lb-ft, N.m</i>
$U$	bending strain energy, <i>lb-ft, N.m</i> Eq.3.9
$U_b$	twisting strain energy, <i>lb-ft, N.m</i> Eq.3.17
$v$	transverse displacement, <i>in, m</i>
$w$	weight per unit length ( $w/l = \rho Ag$ ), <i>lb-ft<sup>-1</sup>, N.m<sup>-1</sup></i>
$W_s$	weight of tube, <i>lb, N</i>
$\Delta W$	sum of work done by the external loads and force, <i>lb-ft, N.m</i>
$W_g$	work against gravity by string <i>lb-ft, N.m</i> Eq.3.16
$W$	tubular weight in mud, <i>lb, N</i>
$x^*$	lock-up depth <i>ft, m</i> Eq.3.69, Eq.3.71
$\alpha$	inclination, <i>degree, radian</i>
$\beta$	buoyancy factor
$d\theta$	dogleg sensitivity, <i>deg/ft</i>
$\varepsilon_a$	axial strain
$\lambda$	slenderness ratio Eq.1.14
$\sigma_y$	yield stress, <i>Nm<sup>-2</sup></i>
$\sigma$	mean stress at failure, <i>Nm<sup>-2</sup></i> Eq. 1.11
$\sigma_a$	Euler critical stress, <i>Nm<sup>-2</sup></i> Eq. 1.8
$\sigma_b$	maximum bending stress, <i>Nm<sup>-2</sup></i> Eq.3.46
$\Delta_b$	tube displacement, <i>ft, m</i> Eq.3.49
$\sigma_a$	axial stress, <i>Nm<sup>-2</sup></i>
$\sigma_r$	radial stress, <i>Nm<sup>-2</sup></i>
$\sigma_\theta$	tangential stresses, <i>Nm<sup>-2</sup></i>
$\sigma_t$	buoyed tensile stress, <i>psi, MPa</i>
$\nu$	Poisson's ratio
$\kappa$	curvature in the vertical plane (build or drop), <i>degree/100ft</i>
$\tau_{r\theta}, \tau$	shear stress, <i>Nm<sup>-2</sup></i> Eq. 4.23
$\sigma_{bE}$	maximum permissible bending stress for grade E pipe, <i>psi, MPa</i>
$\sigma_a$	deviatoric stress <i>psi, MPa</i> Eq.2.8
$\rho_{steel}$	density of steel, <i>lb.ft<sup>-3</sup>, Kg.m<sup>-3</sup></i>
$\rho_{mud}$	density of mud, <i>lb.ft<sup>-3</sup>, Kg.m<sup>-3</sup></i>
$\theta/L$	angle of twist (radians/inch)



## **List of abbreviation**

<i>DLS</i>	dogleg severity, degree/100'
<i>SF</i>	safety factor



## Summary

A continuous application of axial load causes the tubulars to first buckle sinusoidally and then helically. Several models have been proposed for prediction of forces causing helical shape of tubular in vertical wells, inclined and horizontal wells and curved boreholes. However, there is no consensus of which models to be used for better predictions.

The comparison of the existing models with the experimental data shows that the models do not predict all the observed values consistently. The common element among the models is the Dawson Paslay's critical sinusoidal load having different scaling constant  $\chi$ . For instance Chen et al's is (1.414), Wu et. al., (1.828), and Miska/Mitchel (2.828). The inconsistency in the model prediction could be due to the fact that the model assumes constant load and having constant coefficient  $\chi$ . However, the experimental test results have shown that the load is displacement dependent. In this note, considering the laboratory and field observed displacement dependent loading situations; we have regenerated loading history by best least square polynomial fit. Using the fitting coefficients and the analytical model, we observed that the scaling factor coefficient,  $\chi$ , being variable for the various experimental test results. In addition, the turning points of the gradient of the reconstructed loading history correspond to the helical buckling load.

To summarize:

- *It has been reported by the earlier works that the scaling factor,  $\chi$ , being constant for any testing conditions. However, in this work we analyzed that the scaling factor,  $\chi$ , is not constant. It depends on load history and tubular dimensions.*

- *The pitch length determination from Chen, and Miska results some unrealistic values for near-vertical inclined tube. For instance for the well inclination less than 8 degree.*
- *The analysis of Lubiniski's Load-pitch equation is applicable for a vertical tube. When applying for an inclined string, we obtain unrealistic value yielding the pitch length greater than the length of the test string itself. The determinations of pitch length with our analysis utilize all possible energies in the buckled tube. All results show the pitch length lower than the string length itself. The comparison of pitches show that both our analyses and that of Lubiniski's are in agreement for vertical strings. For an inclined string we obtained some difference. Given the fact that Lubiniski's pitch is correct for the vertical string, our pitch analysis is also acceptable. The advantage of our analysis is that it takes into account the effect of well inclination, contrary to Lubiniski.*
- *The applicability of the method-I (section 5.3.2.1) depends on the correctness of the input values (Sinusoidal displacement, Load).*
- *The application of graphical approach, Method-II (section 5.3.2.2 in the main report), determines the buckling load. The method might be promising when running a full scale-buckling test. This method determines helical load, displacement and the helical pitch length.*