

# Analysis, Design, and Optimization of RF CMOS Polyphase Filters

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*To my wife, Hongjin and my son, Berkeley*



# Abstract

Polyphase filters (PPFs) are an efficient solution for high accuracy quadrature generation in radio frequency (RF) CMOS design. Although there are some guidelines for design of RF CMOS PPFs, they give too much freedom. With layout considerations, optimization of RF CMOS PPFs cannot be reached by using analytical calculations because of many constraints and tradeoffs in the design. Thus, in design of RF CMOS PPFs, intensive trials and several design iterations are needed to reach given specifications. In this situation, a computer-aided analysis and design optimization tool for RF CMOS PPFs is of great help to the community of RF CMOS PPFs designers, especially for industrial projects that are under time-to-market pressures. However, such a dedicated tool has not been available until now, to the author's best knowledge. Focusing on computer-aided analysis and design optimization tools for RF CMOS PPFs, this dissertation has the following scientific contributions:

- PPFANA, a dedicated computer-aided software tool for analysis of the influences from the nonideal effects, namely, process tolerance, components mismatch on the quadrature accuracy of the RF CMOS PPFs, was developed by the author and presented in this dissertation.
- Physical design considerations for RF CMOS PPFs in system-on-a-chip (SoC) solution were proposed by the author through the analysis on the influences from parasitic effects in RF SoC.
- Silicon success of an RF CMOS PPF designed by the author for a low-IF Bluetooth receiver using Fraunhofer-IMS 0.6  $\mu\text{m}$  CMOS process was achieved.
- PPFOPTIMA, a computer-aided synthesis and optimization tool for RF CMOS PPFs with dedicated worst-case simulation and Monte Carlo simulation toolboxes for verification of the performance of RF CMOS PPFs, was developed by the author and presented in this dissertation.





# Zusammenfassung

Polyphasenfilter (PPF) sind eine leistungsfähige Lösung für die hochgenaue Quadraturerzeugung im Hochfrequenz (HF)-CMOS-Entwurf. Obgleich es einige Richtlinien für den Entwurf von HF-CMOS-PPF gibt, erlauben diese zu viele Freiheiten. Eine Optimierung der Schaltung und der Layout kann mit Rücksicht auf viele technisch und wirtschaftlich vorgegebene Randbedingungen und der daraus resultierenden Kompromisse mit analytischen Methoden alleine nicht erreichen werden. Es werden aufwändige Versuche und mehrere Entwurfsverbesserungen benötigt, um einen zufriedenstellenden Entwurf zu erreichen. Deshalb ist ein computerbasiertes Werkzeug zur Analyse und Optimierung des Entwurfs von HF-CMOS-PPF besonders für den industriellen Anwendungsbereich wünschenswert. Nach bestem Wissen des Autors gab es bislang ein solches Werkzeug nicht. Im Hinblick auf computerbasierte Werkzeuge zur Analyse und Optimierung des Entwurfs von HF-CMOS-PPF bringt die vorliegende Dissertation folgende wissenschaftliche Beiträge:

- PPFANA, ein spezielles Entwurfswerkzeug für die Simulation der Einflüsse von den nichtidealen Effekten wie beispielsweise von Prozesstoleranzen, von Fehlanpassungen der Komponenten und der Quadraturgenauigkeit, wurde vom Autor entwickelt und vorgestellt.
- Richtlinien für den Entwurf von HF-CMOS-PPF auf „System-on-a-Chip“-Lösungen werden vom Autor unter Berücksichtigung parasitärer Effekte erarbeitet und vorgeschlagen.
- Die erfolgreiche Realisierung eines HF-CMOS-PPF für einen Bluetooth-Empfänger mit niedriger Zwischenfrequenz wurde mit dem Fraunhofer-IMS 0.6  $\mu\text{m}$  CMOS Prozeß nachgewiesen.
- PPFOPTIMA, ein Synthese- und Optimierungswerkzeug für HF-CMOS-PPF auf Basis von Worst-Case- und Monte-Carlo-Simulationen wurde vom Autor entwickelt und vorgestellt.



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Zisan Zhang  
October, 2005  
Duisburg, Germany



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## List of Abbreviations

<b>ABBREVIATION</b>	<b>FULL EXPRESSION</b>
AM	Amplitude mismatch
BALUN	Balance unbalance converter
BiCMOS	Bipolar CMOS
C	Capacitor
CDMA	Code division multiple access
CEC	Compression and error correction
CM	Component mismatch
CMOS	Complementary metal oxide semiconductor
CPU	Central processing unit
DC	Direct current
DCR	Direct-down conversion receiver
DECT	Digitally enhanced cordless telephony
ESD	Electro static discharge
FM	Frequency modulation
GA	Genetic algorithms
GaAs	Gallium arsenide
GM	Gain mismatch
GSM	Global system for mobile communications
GUI	Graphical user interface
IC	Integrated circuit
IF	Intermediate frequency
IMS	Institut für Mikroelektronische Schaltungen und Systeme
I/O	Input / output
I/Q	In phase/quadrature phase
IRF	Image reject filter
IRM	Image reject mixer
IRR	Image reject ratio
ISM	Industrial, scientific and medical

LNA	Low noise amplifier
LO	Local oscillator
LPF	Low pass filter
MC	Monte Carlo
MIM	Metal insulator metal
MLR	Maximum length of resistors
NF	Noise figure
PA	Power amplifier
PC	Personal computer
PCB	Printed circuit board
PD	Phase difference
PDF	Probability density function
PM	Phase mismatching
PPF	Polyphase filter
PPFANA	Polyphase filter analysis toolbox
PPFN	Polyphase filter network
PPFOPTIMA	Polyphase filter optimization toolbox
PS	Postlayout simulation
PT	Process tolerances
QAM	Quadrature amplitude modulation
QM	Quadrature modulation
QPSK	Quadrature phase shift keying
RC	Resistance capacitance
RF	Radio frequency
SA	Simulated annealing
SiGe	Silicon germanium
SoC	System on a chip
VCO	Voltage controlled oscillator
WLAN	Wireless local area network
WCDMA	Wideband code division multiple access

# List of Symbols

$A_I$	Amplitude of the I path output signal in RF CMOS PPFs
$A_Q$	Amplitude of the Q path output signal in RF CMOS PPFs
$A_{R_e}$	Fitting parameter for the standard deviation of $\sigma_{R_e}$
$A_{R_{sh}}$	Fitting parameter for the standard deviation of $\sigma_{R_{sh}}$
$A_{\Delta w}$	Fitting parameter for the standard deviation of $\sigma_{\Delta w}$
$C_C$	Parasitic capacitance between back plate of the capacitor and the substrate
$C_i$	Capacitor of the $i_{th}$ stage in the RF CMOS PPFs
$C_{i,CM}$	Capacitor of the $i_{th}$ stage with component mismatch between I path and Q path in the RF CMOS PPFs
$C_{i,PT}$	Capacitor of the $i_{th}$ stage with process tolerance in the RF CMOS PPFs
$C_{I,i}$	Capacitor of the $i_{th}$ stage in the I path of the RF CMOS PPFs
$C_{para}$	Parasitic capacitance
$C_{Q,i}$	Capacitor of the $i_{th}$ stage in the Q path of the RF CMOS PPFs
$C_R$	Parasitic capacitance between polysilicon resistor and the substrate
$C_{sum}$	Sum of parasitic capacitance between RF CMOS PPFs and the substrate
$C_{uaC}$	Parasitic capacitance between unit area capacitor and the substrate
$C_{uaCi}$	Parasitic capacitance between the capacitors in the $i_{th}$ stage and the substrate
$C_{uaRi}$	Parasitic capacitance between the resistors in the $i_{th}$ stage and the substrate



$C_{\text{unit}}$	Unit capacitance of the capacitor
$d_{\text{phase}}$	Phase difference between I path and Q path in PPFs
$f$	Frequency of a signal
$f_a$	Constant describing influences of area on mismatch
$f_p$	Constant describing influences of periphery on mismatch
$f_d$	Constant describing influences of distance on mismatch
$F_i$	Fitness of the individual $i$
$H(j\omega)$	Transfer function of polyphase filters
$H_I(j\omega)$	Imaginary part of the transfer function of polyphase filters
$H_R(j\omega)$	Real part of the transfer function of polyphase filters
$I_{\text{in}}$	Input current in RF CMOS PPFs
$I_{k,\text{in}}$	The $k_{\text{th}}$ input current in RF CMOS PPFs
$I_{k,\text{out}}$	The $k_{\text{th}}$ output current in RF CMOS PPFs
$I_{\text{out}}$	Output current in RF CMOS PPFs
$L_{\text{max,Res}}$	Maximum length of the resistor
$L_R$	Length of the resistor
$M_C$	Mismatch between the neighboring capacitors
$M_{\text{CMC}}$	Component mismatch of the capacitors
$M_{\text{CMR}}$	Component mismatch of the resistors
$M_R$	Mismatch between the neighboring resistors
$m_{\text{gain,CM}}$	Gain mismatch in RF CMOS PPFs with component mismatch
$m_{\text{gain,ideal}}$	Gain mismatch in the ideal RF CMOS PPFs
$m_{\text{gain,PT}}$	Gain mismatch in RF CMOS PPFs with process tolerances
$m_{\text{phase}}$	Phase mismatch in RF CMOS PPFs
$m_{\text{phase,CM}}$	Phase mismatch in RF CMOS PPFs with component mismatch

$m_{\text{phase,ideal}}$	Phase mismatch in the ideal RF CMOS PPFs
$m_{\text{phase,PT}}$	Phase mismatch in RF CMOS PPFs with process tolerances
$N_{\text{unit}}$	Number of unit capacitor
$P_j$	Probability of selection of the $j$ population
$P_{\text{R}}$	Process tolerance of the resistors
$Q$	Quality factor
$R_{\text{e}}$	End resistance of a resistor
$R_i$	Resistor of the $i_{\text{th}}$ stage in the RF CMOS PPFs
$R_{i,\text{CM}}$	Resistor of the $i_{\text{th}}$ stage with component mismatch between I path and Q path in the RF CMOS PPFs
$R_{\text{I},i}$	Resistors of the $i_{\text{th}}$ stage in the I path of the RF CMOS PPFs
$R_{i,\text{PT}}$	Resistor of the $i_{\text{th}}$ stage with process tolerance in the RF CMOS PPFs
$R_{\text{IR}}$	Image reject ratio
$R_{\text{Q},i}$	Resistors of the $i_{\text{th}}$ stage in the Q path of the RF CMOS PPFs
$R_{\text{sh}}$	Sheet resistance of resistor
$R_{\text{sub}}$	Resistivity of the substrate
$R_{\text{UIR}}$	Ultimately obtainable image reject ratio
$S_{\text{C}_i}$	Area of the capacitor in the $i_{\text{th}}$ stage in RF CMOS PPFs
$S_{\text{R}_i}$	Area of the resistor in the $i_{\text{th}}$ stage in RF CMOS PPFs
$S_{\text{R}}$	Area of a resistor
$V_{\text{in}}$	Input voltage in RF CMOS PPFs
$V_{k,\text{in}}$	The $k_{\text{th}}$ input voltage in RF CMOS PPFs
$V_{k,\text{out}}$	The $k_{\text{th}}$ output voltage in RF CMOS PPFs
$V_{\text{I},\text{in}}$	Real part of the input signal in RF CMOS PPFs
$V_{\text{I},\text{mid}}$	Intermediate I path signal

$V_{Q, \text{mid}}$	Intermediate Q path signal
$V_{\text{Iout}}$	I path output signal in RF CMOS PPFs
$V_{\text{out}}$	Output voltage in RF CMOS PPFs
$V_{Q, \text{in}}$	Imaginary part of the input signal in RF CMOS PPFs
$V_{Q\text{out}}$	Q path output signal in RF CMOS PPFs
$W_R$	Width of a resistor
$w_i$	Weight of the $i_{\text{th}}$ item
$Y$	Admittances
$\omega$	Angular frequency
$\omega_{\text{IF}}$	Angular frequency of the intermediate frequency signal
$\omega_{\text{image}}$	Angular frequency of the image
$\omega_{\text{LO}}$	Angular frequency of the signal from the local oscillator
$\omega_{\text{RF}}$	Angular frequency of the radio frequency signal
$\omega_{\text{signal}}$	Angular frequency of the signal
$\sigma_a$	Standard deviation of mismatch caused by area variation
$\sigma_{\text{component}}$	Standard deviation of component mismatch
$\sigma_d$	Standard deviation of mismatch caused by distance
$\sigma_p$	Standard deviation of mismatch caused by periphery
$\sigma_{\text{Re}}$	Standard deviation of the end resistance
$\sigma_{R_{\text{sh}}}$	Standard deviation of the sheet resistance
$\sigma_{\Delta w}$	Standard deviation of the width of a resistor
$\phi$	Phase of a signal
$\theta$	Relative phase difference in RF CMOS PPFs
$\Delta A$	Amplitude mismatch in volts
$\Delta\phi$	Phase mismatch in degrees