

**Reliability Information in Channel Decoding**  
**Practical Aspects and**  
**Information Theoretical Bounds**

**Dissertation**

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# Preface

This thesis was written during my time as a research and teaching assistant at the Information and Coding Theory Lab, Faculty of Engineering, University of Kiel, Germany.

I wish to express my most sincere gratitude to my advisor Prof. Dr. Peter Adam Höher. I would like to thank him for all the inspiring discussions we had; for giving me scientific freedom which allowed me to develop my own ideas; and for introducing me to the scientific community. His scientific mind and his enthusiasm are reflected in this work.

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A handwritten signature in dark ink, appearing to read 'Ingmar Land'. The script is cursive and fluid, with the first name 'Ingmar' written in a larger, more prominent style than the last name 'Land'.

Ingmar Land

Kiel, May 2005



# Abstract

This thesis addresses the use of reliability information in channel decoding and covers practical aspects as well as information-theoretical bounds. The considered transmission systems comprise linear binary channel encoders, symmetric memoryless communication channels, and non-iterative or iterative symbol-by-symbol soft-output channel decoders.

The notions of accurate and mismatched reliability values are introduced, and the measurement and improvement of the quality of reliability values are discussed. A criterion based on the Kullback-Leibler distance is proposed to assess the difference between accurate and mismatched reliability values. The concepts are applied to iterative decoders for parallel concatenated codes.

Accurate reliability values may be exploited to estimate transmission quality parameters, such as the bit error probability or the symbol-wise mutual information between encoder input and decoder output. The proposed method is unbiased and does not require knowledge of the transmitted data. A general framework for this kind of estimation is introduced, and the advantage of the proposed method over the conventional method is shown analytically by comparing the estimation variances. The proposed method may be used for a “blind” estimation at the receiver side or to speed up the estimation in simulations.

Symbol-by-symbol soft-output decoding may be interpreted as processing of mutual information. Assuming accurate reliability values at the input and at the output of a decoder, its decoding behavior may be characterized by information transfer functions, such as information processing characteristics (IPCs) or extrinsic information transfer (EXIT) functions. Bounds on information transfer functions are derived using the concept of bounding combined information, which is formed by combining single values of mutual information with respect to code constraints. These bounds are valid for all binary-input symmetric memoryless channels, and thus no Gaussian assumption is required, as in the original EXIT chart method. Single parity-check codes, repetition codes, and the accumulator are addressed. Based on such bounds, decoding thresholds for low-density parity-check codes are analytically determined.

**Keywords:** Linear binary codes, parallel concatenated codes (PCCs), serially concatenated codes (SCCs), low-density parity-check codes (LDPCs), symbol-by-symbol soft-output decoding, iterative decoding, reliability information, parameter estimation, information processing characteristics (IPC), extrinsic information transfer (EXIT) functions, EXIT charts, information combining, bounds on information combining.





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