ON THE ANALYSIS OF NONLINEAR ELECTROMECHANICAL SYSTEMS WITH APPLICATIONS

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ABSTRACT

Index Terms: Coupled Nonlinear Oscillators; Electromechanical Transducer; Equilibrium Points; Stability; Local Bifurcations; Oscillatory States; Chaos; Bifurcations; Analogue Simulation.

This dissertation studies the dynamics of a physical system consisting of a self-sustained oscillator of the **Van der Pol** type coupled to an anharmonic oscillator of the **Duffing** type and discusses some applications of the coupled system.

The interest devoted to this particular type of coupled system is due to its technological and fundamental applications. Indeed, the coupling between the **Van der Pol** and **Duffing** type oscillators can describe the behaviour of a self-sustained electromechanical transducer, which consists of an electrical part described by the **Van der Pol** oscillator coupled to a mechanical part described by the **Duffing** oscillator. The rich and complex dynamical modes exhibited by the coupled system can be used in laboratory applications for measurement and instrumentation. The chaotic behaviour exhibited by the coupled system can also be used in the field of telecommunications, for instance in secure communication or in chaos-based modulation schemes.

The main accomplishment of this research can be explained as follows. In the industry and even in the consumer world, diverse electromechanical systems are intensively used to perform various tasks. These systems do realize a coupling of two types of oscillating systems. The mechanical part can be modelled as a **Duffing** oscillator type, whereas the electrical part can be modelled as a **Van der Pol** oscillator type.

Besides, it is known that certain parameter settings of coupled systems do result in very negative consequences on the system's performance. The so-called resonance is a good example, which shows that even the system's ruin can be provoked. One distinguishes three types of functioning zones related to the parameter setting space: the regular zone(s), the irregular or chaotic zone(s), and the domain(s) in which quenching phenomena (that is, the death of oscillations) are observed. For both a good dimensioning of the electromechanical systems and an optimal real-time control of such systems, one needs a set of theoretical and methodical tools that will allow the system designer to identify and subsequently avoid the system parameter settings leading to the unwished functioning zones. The present research, which consists the systematic study of the non-linear dynamic of the coupling between the **Van der Pol** and **Duffing** oscillators of provides for the first time, in a solid way – since validated by extensive experimental analysis, those theoretical and methodical tools which will be of precious use for design and control engineers of modern electromechanical systems. This is even more crucial as some of these systems will progressively become miniaturized (one generally refers to it as mechatronics).

Chapter I deals with the general introduction in which the choice devoted to this research topic is justified. Some generalities on nonlinear oscillators are presented, followed by a description of the oscillators of type **Van der Pol** and **Duffing**. The chapter ends by both a justification of the methods (approaches or techniques) used in our investigations and a presentation of the headlines of the work in this dissertation.

Chapter II studies the elastic coupling (coupling through the solution) and the gyroscopic coupling (coupling through the acceleration) between the **Van der Pol** and **Duffing** oscillators. The stability of the equilibrium points is analysed using the **Routh-Hurwitz** criterion. Their local bifurcations are investigated and different types of

bifurcations likely to appear in the coupled system are obtained. Analytic conditions for the appearance of bifurcations of the **Hopf** type are derived. Analytic oscillatory solutions are obtained in both resonant and non-resonant cases. Both the autonomous and the non-autonomous states of the coupled system are considered. A particular interest is devoted to the behaviour of the coupled system in the state of internal resonance. Hysteresis, quenching and beating phenomena are observed. **Shilnikov's** theorem for three-dimensional dynamical system is used to define the range of the coupling coefficients corresponding to the chaotic domain. A numerical computation is carried out to verify the theoretical predictions and to study the transitions in the states of the coupled system. The bifurcation diagrams associated to the largest one-dimensional (1-D) numerical **Lyapunov** exponent are obtained showing period-adding, period-doubling, torus breakdown and sudden transition routes to chaos. Also shown is the extreme sensitivity of the coupled system to both initial conditions and tiny changes in its parameters. Basins of attraction are obtained showing the effects of the initial conditions on the behaviour of the coupled system. An experimental study is carried out to verify the theoretical and numerical results. An appropriate electronic circuit (analogue simulator) using analogue devices such as multipliers AD633JN and operational amplifiers LF351 is proposed. Experimental amplitude- and frequency-response curves are obtained in the case of internal resonance. Various bifurcation points are observed following the variations of both the coupling coefficients and the amplitude of the excitation. Distinct routes to chaos are observed: sudden transition, period- doubling and period- adding. A comparison of the experimental and numerical results with the theoretical analysis shows an interesting agreement.

Chapter III considers both the elastic coupling and the dissipative coupling (coupling through the velocity) between the **Van der Pol** and **Duffing** oscillators. The stability of the equilibrium points is analysed using the **Routh-Hurwitz** criterion. A new

technique based on the multiple time scale method is proposed for the investigation of the stability of the critical equilibrium point $\left(x,\ \frac{dx}{dt},\ y,\ \frac{dy}{dt}\right)=\left(0,\ 0,\ 0,\ 0\right)$. Analytic oscillatory solutions are obtained in both the resonant and non-resonant cases. Hysteresis, quenching and beating phenomena are observed in the state of internal resonance. Digital computation is carried out to verify the theoretical predictions and to study the transitions in the states of the coupled system. The bifurcation diagrams associated to the largest one-dimensional numerical **Lyapunov** exponent are obtained showing period-adding, period-doubling, torus breakdown and sudden transition routes to chaos. It is also shown the extreme sensitivity of the coupled system to small changes in its parameters. An experimental study is carried out to verify the theoretical and numerical results. An appropriate electronic circuit (analogue simulator) using multipliers AD633JN and operational amplifiers LF351 is proposed. Experimental frequency-response curves are obtained in the case of internal resonance. Various bifurcation points are observed following the variations of the natural frequency of the **Duffing** oscillator. Distinct routes to chaos are observed: sudden transitions, period-doubling and period-adding transitions. A comparison of the experimental and numerical results with the theoretical analysis shows a very good agreement.

One of the most important contributions of this work is to provide a set of reliable analytical expressions (formulae) describing the coupled system behaviour. These analytical formulae are of great importance for design engineers. Their reliability is demonstrated by a very good agreement with the results obtained from both the numerical and experimental analysis.

The last chapter (Chapter IV) is devoted to the general conclusion. Summary of the results and the discussions of some applications of the coupled system are presented.

New and interesting problems (or issues) are formulated. Future research focussing on these issues will complete and provide a better understanding of the work tackled in this dissertation.

Zusammenfassung der Arbeit

Schlüsselwörter

Der nicht-lineare gekoppelte Oszillator; der elektromechanische Transduktor (oder Wandler); die Gleichgewichtspunkte; die Stabilität; die Schwingungszustände; das Chaos; die Verzweigungen; die analoge Simulation.

Diese Dissertation befasst sich mit der Dynamik eines physikalischen Systems bestehend aus einem sich selbstaufrechterhaltenden (nichtlinearen) Oszillator vom Typ **Van der Pol,** gekoppelt zu einem anderen nicht-harmonischen Oszillator vom Typ **Duffing.** Des weiteren wird die industrielle Anwendung des gekoppelten Systems diskutiert.

Das Interesse an diesem speziellen Typ eines gekoppelten Systems liegt an dessen Anwendbarkeit bzw. Einsatz, zum einen, in technologischen Anwendungen und, zum anderen, in Grundlagenforschung. Die Kopplung eines Oszillators vom Typ **Van der Pol** mit einem vom Typ **Duffing** kann in der Tat das Verhalten eines sich selbstaufrechterhaltenden elektromechanischen Wandlers beschreiben, welcher aus einem elektrischen Teil vom Oszillatortyp **Van der Pol** und einem mechanischen Teil vom Oszillatortyp **Duffing** besteht. Die zahlreichen und komplexen dynamischen Modi des gekoppelten Systems können in diversen Laboranwendungen der Messtechnik genutzt werden. Das chaotische Verhalten des gekoppelten Systems kann ebenfalls in der

Fernmeldetechnik verwendet werden, insbesondere in den Gebieten *Verschlüsselung von Kommunikationsströmen* oder in *Chaos-basierten Modulationsansätzen*.

Das Kapitel 2 befasst sich sowohl mit der elastischen als auch mit der gyroskopischen Kopplung zwischen einem Oszillator vom Typ **Van der Pol** und einem vom Typ **Duffing**. Die Stabilität der Gleichgewichtspunkte wird mit Hilfe des **Routh-Hurwitz** Kriteriums analysiert. Die analytischen Schwingungslösungen für sowohl den Resonanz- als auch für den Nicht-Resonanzfall sind ermittelt worden. Man betrachtet, neben den autonomen Zuständen, auch die nicht-autonomen. Ein besonderes Augenmerk wird dem Verhalten des gekoppelten Systems im Zustand der internen Resonanz gewidmet. Folgende Phänomene werden hierbei betrachtet: Hysterese, Quenching (Dämpfung) und Beating (Anschlag). Das Theorem von **Shilnikov** für drei-dimensionale dynamische Systeme kann genutzt werden, um die Größenordnung der Kopplungskoeffizienten zu bestimmen, die dem chaotischen Bereich zuzuordnen sind. Eine numerische Berechnung wird gebraucht, zum einen, um eine Überprüfung der theoretischen Ergebnisse vorzunehmen, und zum anderen, um die Zustandsübergänge des gekoppelten Systems zu beleuchten. Die Bifurkationsdiagramme, zusammengenommen mit dem größten, ein-dimensionalen numerischen **Lyapunov-**Exponenten, werden ermittelt und zeigen eine Reihe von Übergangsrouten zum Chaos: *‚period-adding transition', ‚period-doubling transition', ‚torus breakdown transition'* und *‚sudden transition'*. Die extreme Empfindlichkeit des gekoppelten Systems sowohl zu Ausgangsbedingungen als auch zu Parameterveränderungen ist ebenfalls beobachtet worden. Attraktionsbecken (*basins of attraction*) konnten ermittelt werden, welche die Auswirkung der Ausgangsbedingungen auf das Systemverhalten klar darstellen. Eine experimentelle Studie wurde durchgeführt, um sowohl die theoretischen als auch die numerischen Lösungen zu überprüfen. Eine angepasste elektronische Schaltung (Analoger Simulator) wurde vorgeschlagen und aufgebaut. Diese Schaltung besteht im wesentlichen aus analogen Bauteilen wie einem Multiplizierer AD633JN und Operationsverstärkern LF351. Experimentelle Kurven, die den Verlauf von Amplitude und Frequenz wiedergeben, sind für den Fall der internen Resonanz erfasst worden. Diverse Bifurkationspunkte konnten beobachtet werden, sowohl für die Kopplungskoeffizienten als auch für die Amplitude der Anregungen (*excitation amplitude*). Unterschiedliche Wege oder Routen zum Chaos werden beobachtet: *sudden transition, period- doubling and period- adding*. Ein Vergleich der Ergebnisse aus den drei Lösungsansätzen (d.h. analytischer, numerischer und experimenteller) zeigt eine sehr gute Übereinstimmung zwischen allen dreien.

Das Kapitel 3 betrachtet sowohl die elastische als auch die dissipative Kopplung (Kopplung durch Geschwindigkeit) zwischen Oszillatoren vom Typ **Van der Pol** und denen vom Typ **Duffing**. Das **Routh-Hurwitz**-Kriterium wird angewandt, um die Stabilitätsanalyse der Gleichgewichtspunkte vorzunehmen. Eine neue Technik, basierend auf den sogenannten „*multiple time scales*“, wird für die Stabilitätsanalyse des folgenden, kritischen Gleichgewichtspunktes vorgeschlagen: $\left(x,\ \frac{dx}{dt},\ y,\ \frac{dy}{dt}\right) = (0,\ 0,\ 0,\ 0)$. Die analytischen Lösungen sowohl für den Resonanz- als auch für den Nicht-Resonanz-Fall sind ermittelt worden. Die Phänomene *Hysterese, Quenching und Beating* werden im Zustand der internen Resonanz betrachtet. Eine numerische Berechnung bzw. Simulation ist, erstens, zur Überprüfung der theoretischen Schätzungen, und zweitens, zur Studie der Zustandsübergänge des gekoppelten Systems durchgeführt worden. Die Bifurkationsdiagramme, zusammengenommen mit dem größten ein-dimensionalen numerischen **Lyapunov**-Exponenten sind ermittelt und zeigen diverse Wege bzw. Routen zum Chaos: *period-adding, period-doubling, torus breakdown and sudden transition*. Gezeigt wird ebenfalls die extreme Empfindlichkeit des gekoppelten Systems zu leichten Variationen seiner Parameter. Eine experimentelle Studie wird durchgeführt, welche der Verifizierung sowohl der theoretischen (bzw. analytischen) als auch der numerischen Ergebnisse dient. Eine angepasste elektronische Schlatung (Analoger Simulator) wurde

vorgeschlagen und aufgebaut. Diese benutzt im wesentlichen analoge Komponenten wie der Multiplizierer AD633JN und Operationsverstärker des Typs LF351.
Experimentelle Kurven der Frequenzantwort werden für den Fall der internen Resonanz erfasst. Diverse Bifurkationspunkte werden ebenfalls beobachtet, welche den Variationen der natürlichen Frequenz des Duffing-Oszillators folgen. Unterschiedliche Routen zum Chaos sind ausgemacht worden: *sudden transitions, period-doubling and period-adding transitions*. Außerdem, ein Vergleich von experimentellen Ergebnissen zu sowohl theoretischen als auch numerischen Ergebnissen zeigt eine sehr gute Übereinstimmung.

Das letzte Kapitel enthält, neben einer Zusammenfassung der wichtigsten Ergebnisse, einige wenige industrielle Anwendungen des gekoppelten Systems. Neue interessante Fragestellungen werden formuliert. Die zukünftige Forschung, die sich mit diesen neuen Fragen befassen wird, wird die in dieser Dissertation präsentierte Arbeit ergänzen und zu deren besserem Verständnis beitragen.

DEDICATION

To **FONKOU Oumbé André** (my late Father) and **MATSINGANG Pauline** (my late Mother)

To **TANKAM Jacqueline** (my late Twin-Sister)

To **DONFACK Chrétien** (my late father-in-law) and **METANGO Bernadette** (my Mother-in-law)

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Hannover *Jean Chamberlain Chedjou*

Symbols

Symbol	Meaning
$\mid\ \mid$	Absolute value
$\dot{} = \frac{d}{dt}$	Single over dot / First derivative
$\ddot{} = \frac{d^2}{dt^2}$	Double over dot / Second derivative
t	time variable
$\rightarrow$	Transition from left to right
$<$	Less than
$>$	Greater than
$<<$	Far less than
$>>$	Far greater than
$\int$	Continuous summation
$\sum$	Discrete summation
$\in$	Appurtenance
[x]	Reference number x
$\sqrt{\ }$	Square root
i	$\sqrt{-1}$
f	Frequency measured in Hz
$\omega = 2\pi f$	Angular frequency (that is Frequency measured in $\frac{rad}{s}$)
$\omega t + \theta$	Total phase of a waveform

θ	Initial phase of a waveform
$T = \dfrac{2\pi}{\omega}$	Period of the waveform
σ	A detuning parameter
ε	Small dimensionless parameter
$\approx$	Almost equal
$x(t)$ and $y(t)$	Waveforms solution
$\dfrac{dx}{dt}$	Velocity
$\dfrac{d^2x}{dt^2}$	Acceleration
$\times$	Cross Product

Acronyms

Lim	Limit
AD	Analogue device
Eq.	Equation
Fig.	Figure
c.c.	Complex conjugate
$X_{\max}$	Maximal value of X
HF	High frequency
VHF	Very high frequency
VLF	Very low frequency
et al.	And others
Ln	Logarithm function
Exp	Exponential function
Cos	Cosine function
Sin	Sine function
R	Resistor
C	Capacitor
L	Inductor
RNL	Nonlinear resistor
DFT	Discrete Fourier Transform
I_d	Identity matrix
M_J	Jacobian Matrix
1-D	One-dimensional
$\lambda_{\max}$	Largest One-dimensional numerical **Lyapunov** exponent
BJT	Bipolar Junction Transistor

KCL	**Kirchhoff** Current Law
KVL	**Kirchhoff** Voltage Law
NDL	**Newton** Dynamics Law

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