Congruent Numbers

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2004

Berichte aus der Mathematik

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Shaker Verlag
Aachen 2004

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the internet at http://dnb.ddb.de.

Printed in Germany.

ISBN 3-8322-2684-2
ISSN 0945-0882

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen
Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9
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Preface

In number theory, Pythagorean Triples PT and congruent numbers are closely connected. The latter are natural numbers A which can be deduced by rational PTs $a,b,c \in Q^+$ whose area is $Aq^2=ab/2$ with $A,q \in N$. They are known since ancient times and a matter of research also in our days. The formulae for their construction are known at least since Euclid, but the decision whether a given number is a congruent number and especially its representation are not so simple.

In this book some general characteristics and theorems are discussed. For the longer proofs, references are given.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in January 2004 Uwe Kraeft

Choice of symbols

$\Rightarrow$	by this follows
$\forall$	for all
$\exists$	there is/are
$\in$	is element of (is contained in)
$\subset$	is subset of (all elements are contained in)
$\cup$, $\cap$	union and intersection of sets
$\varnothing$	the empty set
A={a,b,c}	an example of a set A with elements a, b, and c
# {...}	number of elements
a, α ...	in this text mainly natural numbers or integers
N	set of natural numbers 1, 2, 3, ...
N^-	={-N}={-n; n$\in$N}, set of negative integers -1, -2, -3, ...
N^0	N$\cup${0}
P	primes of N
P^1	P$\cup${1}, primes P included 1
Z	=N$\cup${N^-}$\cup${0}, set of integers
Q	set of rational numbers a/b with a$\in$Z, b$\in$N
Q^+	set of positive rational numbers a/b with a,b$\in$N
R	set of real number algorithms
Q(R)	Q or R
C	complex numbers x+yi with x,y$\in$Q(R)
[r,s]	closed interval with r$\le$t$\le$s and r,s,t$\in$Q
$\cong$	so near as you want but not identical
$\equiv$	$a\equiv b \pmod c \Leftrightarrow a\equiv b_c \Leftrightarrow (a-b)/c\in Z$ for a,b$\in$Z, c$\in$N
(a<b)$\in$Q	a<b and both are elements of Q

f.e. for example (e.g.)

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