

Congruent Numbers

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Berichte aus der Mathematik

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Preface

In number theory, Pythagorean Triples PT and congruent numbers are closely connected. The latter are natural numbers A which can be deduced by rational PTs $a, b, c \in \mathbb{Q}^+$ whose area is $Aq^2 = ab/2$ with $A, q \in \mathbb{N}$. They are known since ancient times and a matter of research also in our days. The formulae for their construction are known at least since Euclid, but the decision whether a given number is a congruent number and especially its representation are not so simple.

In this book some general characteristics and theorems are discussed. For the longer proofs, references are given.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in January 2004

Uwe Kraeft

Choice of symbols

\Rightarrow	by this follows
\forall	for all
\exists	there is/are
\in	is element of (is contained in)
\subset	is subset of (all elements are contained in)
\cup, \cap	union and intersection of sets
\emptyset	the empty set
$A=\{a,b,c\}$	an example of a set A with elements a, b, and c
$\# \{...\}$	number of elements
$a, \alpha \dots$	in this text mainly natural numbers or integers
\mathbb{N}	set of natural numbers 1, 2, 3, ...
\mathbb{N}^-	$=\{-\mathbb{N}\}=\{-n; n \in \mathbb{N}\}$, set of negative integers -1, -2, -3, ...
\mathbb{N}^0	$\mathbb{N} \cup \{0\}$
\mathbb{P}	primes of \mathbb{N}
\mathbb{P}^1	$\mathbb{P} \cup \{1\}$, primes \mathbb{P} included 1
\mathbb{Z}	$=\mathbb{N} \cup \{\mathbb{N}^-\} \cup \{0\}$, set of integers
\mathbb{Q}	set of rational numbers a/b with $a \in \mathbb{Z}$, $b \in \mathbb{N}$
\mathbb{Q}^+	set of positive rational numbers a/b with $a, b \in \mathbb{N}$
\mathbb{R}	set of real number algorithms
$\mathbb{Q}(\mathbb{R})$	\mathbb{Q} or \mathbb{R}
\mathbb{C}	complex numbers $x+yi$ with $x, y \in \mathbb{Q}(\mathbb{R})$
$[r,s]$	closed interval with $r \leq t \leq s$ and $r, s, t \in \mathbb{Q}$
\cong	so near as you want but not identical
\equiv	$a \equiv b \pmod{c} \Leftrightarrow a \equiv b_c \Leftrightarrow (a-b)/c \in \mathbb{Z}$ for $a, b \in \mathbb{Z}$, $c \in \mathbb{N}$
$(a < b) \in \mathbb{Q}$	$a < b$ and both are elements of \mathbb{Q}

f.e. for example (e.g.)

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