# Topological Number Theory

**Uwe Kraeft**

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Berichte aus der Mathematik

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## Preface

Topology is that part of geometry which shows closest relations to numbers. The very beginning of this mathematical discipline was perhaps the middle of the 19th century. Today, there exist several good textbooks. Nevertheless, topology of numbers remains also nowadays a matter of discussions and field of research. This text is no introduction to topology but shows only that special part which deals with the application in number theory.

In chronological order with former books on Pythagorean Triples, Euclidean Sequences, Diophantine Equations, Archimedean Approximations, Number Theory of Adjunctions, Statistical Number Theory, and Arithmetic Number Theory this eighth text is a basic treatment of topological number theory and written for all who are interested in basic mathematics. It is also a major aim to show the methods used in number theory. Therefore the reader can find in the here told proofs more syllogisms than usual in textbooks. Especially this may be of interest for the student who begins to study the elements of number theory. The book is dedicated to the memory of August Ferdinand Möbius, whose ideas led to modern topology, and to Arthur Moritz Schoenflies, who found separately from Evgraph Stepanovich Fedorov the 230 3-dimensional space groups.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in May 2003 Uwe Kraeft

**Choice of symbols**

| | |
|---|---|
| $\Rightarrow$ | by this follows |
| $\forall$ | for all |
| $\exists$ | there is/are |
| $\in$ | is element of (is contained in) |
| $\subset$ | is subset of (all elements are contained in) |
| $\cup$, $\cap$ | union and intersection of sets |
| $\varnothing$ | the empty set |
| A={a,b,c} | an example of a set A with elements a, b, and c |
| {{a,a,b}} | an example of an assemblage with elements a, a, and b |
| $\complement A$ | $=\{x\in X; x\notin A\}$, the complement of the set $A\subset X$ |
| **P**(A) | set of all subsets of A |
| **U**(t) | set of all neighbourhoods of the point t |
| **I**(A) | interior of A (see chapter 9) |
| $\overline{A}$ | closed envelope of A (see chapter 9) |
| **B** | basis (see chapter 9) |
| $\mathbb{B}$ | filter basis (see chapter 9) |
| $\mathbb{F}$ | filter (see chapter 9) |
| f | mapping |
| f(x) | map of the original x |
| $f^{-1}$ | inverse mapping |
| a, $\alpha$ ... | in this text mainly natural numbers or integers |
| N | set of natural numbers 1, 2, 3, ... |
| $N^-$ | $=\{-N\}=\{-n; n\in N\}$, set of negative integers -1, -2, -3, ... |
| $N^0$ | $N\cup\{0\}$ |
| P | primes of N |
| $P^1$ | $P\cup\{1\}$, primes P included 1 |
| Z | $=N\cup\{N^-\}\cup\{0\}$, set of integers |
| Q | set of rational numbers a/b with $a\in Z$, $b\in N$ |
| $Q^+$ | set of positive rational numbers a/b with $a,b\in N$ |
| R | set of real number algorithms |
| Q(R) | Q or R |
| C | complex numbers x+yi with $x,y\in Q(R)$ |
| [-r,s] | closed interval with $-r\le t\le s$ and $r,s,t\in Q$ |
| $\cong$ | so near as you want but not identical |
| $(a<b)\in Q$ | a<b and both are elements of Q |

**Content**