# Fast Numerical Techniques in Computational Electromagnetics for Planar-Stratified Media

Norbert Geng

**HABILITATIONSSCHRIFT**

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| | Prof. Dr. Lawrence Carin, Duke University, NC, USA |
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**Fakultät für Elektrotechnik und Informationstechnik**
**Universität (TH) Karlsruhe**

Berichte aus der Hochfrequenztechnik

**Norbert Geng**

# Fast Numerical Techniques in Computational Electromagnetics for Planar-Stratified Media

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# Vorwort

Die vorliegende Habilitationsschrift ist eine Zusammenfassung der wesentlichen Ergebnisse meiner Arbeiten zu schnellen feldtheoretischen Verfahren in der Radartechnik. Ideale Voraussetzungen für eine intensive Beschäftigung mit diesem hochinteressanten Themenkomplex hatte ich während eines 18-monatigen Post-Doc-Aufenthalts an der Duke University, Durham, North Carolina, USA von Januar 1997 bis Juli 1998 sowie als wissenschaftlicher Mitarbeiter am Institut für Höchstfrequenztechnik und Elektronik der Universität Karlsruhe in der darauf folgenden Zeit bis Dezember 1999.

Mein besonderer Dank gilt an dieser Stelle Prof. Dr. Lawrence (Larry) Carin vom Department of Electrical and Computer Engineering der Duke University. Ich hatte das große Glück, während meiner Zeit als Post-Doc von seinem umfassenden theoretischen Wissen zu profitieren und zu lernen. Es freut mich ganz besonders, daß die 18 Monate Duke University nicht nur eine Reihe von Publikationen, sondern insbesondere auch die Freundschaft zu Larry sowie seinen PhD-Student(inn)en und Post-Docs Andy, Abhijeet, Bimba, Eric, Mark, Traian, Paul und Nilanjan hervorbrachte. Ganz besonders danken möchte ich Dr. Anders (Andy) Sullivan, der alle umfangreichen Berechnungen auf dem Parallel-Supercomputer der US Army/AirForce durchführte, auf die ich als Nicht-US-Staatsbürger keinen unmittelbaren Zugriff hatte.

Mein Dank gilt aber nicht zuletzt Prof. Dr.-Ing. Werner Wiesbeck, dem Leiter des Instituts für Höchstfrequenztechnik und Elektronik (IHE) an der Universität Karlsruhe, der im Sommer 1996 im Rahmen der IEEE Antennas and Propagation Konferenz in Baltimore, Maryland, USA den Kontakt zu Larry herstellte und mich vor, während und nach dem USA-Aufenthalt stets optimal unterstützte und förderte.

Neben Prof. Dr. Werner Wiesbeck und Prof. Dr. Lawrence Carin darf ich mich auch ganz herzlich bei Prof. Dr.-Ing. Rüdiger Vahldieck von der ETH Zürich, Schweiz für die Durchsicht dieser Habilitationsschrift und die Übernahme eines der Referate bedanken.

Taufkirchen bei München, im März 2001

Norbert Geng

# Contents

# Frequently Used Symbols and Abbreviations

Symbols or abbreviations not listed here are defined locally in the text.

## Mathematical Notation

| | |
|---|---|
| $A$ | real or complex scalar |
| $A'+jA''$ | real ($A'$) and imaginary ($A''$) part of a complex scalar |
| $A^*$ | complex-conjugate of (scalar or vector) quantity |
| $\mathbf{A}$ | real or complex vector |
| $\mathbf{A}\cdot\mathbf{B}$ | scalar product of two vectors |
| $\mathbf{A}\mathbf{B}$ | dyadic product of two vectors |
| $\mathbf{A}\times\mathbf{B}$ | cross product of two vectors |
| $\mathbf{A}*\mathbf{B}$ | convolution of two (scalar or vector) functions |
| $\langle\mathbf{A},\mathbf{B}\rangle$ | symmetric product of two vector functions |
| $(\mathbf{A},\mathbf{B})$ | inner (or scalar) product of two vector functions |
| $\overleftrightarrow{\mathbf{I}}$ | unit dyadic $\overleftrightarrow{\mathbf{I}} = \hat{\mathbf{x}}\hat{\mathbf{x}}+\hat{\mathbf{y}}\hat{\mathbf{y}}+\hat{\mathbf{z}}\hat{\mathbf{z}} = \hat{\mathbf{r}}\hat{\mathbf{r}}+\hat{\theta}\hat{\theta}+\hat{\phi}\hat{\phi} = \hat{\rho}\hat{\rho}+\hat{\phi}\hat{\phi}+\hat{\mathbf{z}}\hat{\mathbf{z}}$ |
| $\overleftrightarrow{\mathbf{A}}$ | dyadic (e.g., $\overleftrightarrow{\mathbf{A}}=A^{xx}\hat{\mathbf{x}}\hat{\mathbf{x}}+A^{xy}\hat{\mathbf{x}}\hat{\mathbf{y}}+A^{xz}\hat{\mathbf{x}}\hat{\mathbf{z}}+\ldots+A^{zz}\hat{\mathbf{z}}\hat{\mathbf{z}}$ if Cartesian coordinates) |
| $[\mathbf{A}]$ | real or complex matrix |
| $[\mathbf{A}]^{\mathsf{T}}$ | transpose of a (real or complex) matrix $[\mathbf{A}]$ |
| $[\mathbf{A}]^{\dagger}$ | complex-conjugate transpose $[\mathbf{A}]^{\dagger}=[\mathbf{A}]^{\mathsf{T}*}$ of a complex matrix $[\mathbf{A}]$ |
| $(x,y,z)$ | Cartesian coordinates |
| $(\rho,\phi,z)$ | cylindrical (polar) coordinates |
| $(r,\theta,\phi)$ | spherical coordinates |
| $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ | unit vectors in Cartesian coordinates |
| $\hat{\rho}$, $\hat{\phi}$, $\hat{\mathbf{z}}$ | unit vectors in cylindrical (polar) coordinates |
| $\hat{\mathbf{r}}$, $\hat{\theta}$, $\hat{\phi}$ | unit vectors in spherical coordinates |
| $\mathrm{Re}(\mathbf{A})$ | real part of a complex (scalar or vector) quantity |
| $\mathrm{Im}(\mathbf{A})$ | imaginary part of a complex (scalar or vector) quantity |
| $\partial/\partial\mathrm{n}$ | normal derivative |
| $\nabla$ | Nabla operator (e.g., $\nabla = \hat{\mathbf{x}}\,\partial/\partial x + \hat{\mathbf{y}}\,\partial/\partial y + \hat{\mathbf{z}}\,\partial/\partial z$ if Cartesian coordinates) |
| $\tilde{\nabla}$ | Nabla in spectral domain (e.g., $\tilde{\nabla}=-jk_x\hat{\mathbf{x}}-jk_y\hat{\mathbf{y}}+\hat{\mathbf{z}}\,\partial/\partial z$ if Cartesian coordinates) |
| $\nabla'$ | Nabla operating on source coordinates (e.g., $\nabla=\hat{\mathbf{x}}\,\partial/\partial x'+\hat{\mathbf{y}}\,\partial/\partial y'+\hat{\mathbf{z}}\,\partial/\partial z'$) |
| $\nabla A$ | gradient of a scalar function $A(\mathbf{x})$ |
| $\nabla\times\mathbf{A}$ | curl of a vector function $\mathbf{A}(\mathbf{x})$ |
| $\nabla\cdot\mathbf{A}$ | divergence of a vector function $\mathbf{A}(\mathbf{x})$ |

| | |
|---|---|
| $\nabla_s \cdot \mathbf{A}$ | surface divergence $(\nabla - \hat{\mathbf{n}}\partial/\partial \mathrm{n})\cdot\mathbf{A}$ of a vector function $\mathbf{A}(\mathbf{x})$ |
| $\tilde{\mathbf{A}}$ | two-dimensional spatial Fourier transform $\tilde{\mathbf{A}}(k_x,k_y,z)$ of a function $\mathbf{A}(x,y,z)$ |
| $\delta(\ldots)$ | Dirac impulse |
| $\mathcal{F}\{\ldots\}$ | Fourier transform |
| $\mathcal{F}_s\{\ldots\}$ | shifted Fourier transform |
| $\mathcal{F}^{-1}\{\ldots\}$ | inverse Fourier transform |
| $h_l^{(2)}(\ldots)$ | spherical Hankel function of the second kind and order $l$ |
| $j_l(\ldots)$ | spherical Bessel function of the first kind and order $l$ |
| $J_m(\ldots)$ | Bessel function of the first kind and order $m$ |
| $\mathcal{L}\{\ldots\}$ | general (integro-differential) operator |
| $\mathrm{O}(x)$ | complexity is on the order of $x$ |
| $P_l(\ldots)$ | Legendre polynomial |
| $\mathcal{S}_n\{\ldots\}$ | Sommerfeld-type integral |

## General Abbreviations

| | |
|---|---|
| 2D | two-dimensional |
| 3D | three-dimensional |
| ABC | absorbing boundary condition |
| AIM | adaptive integral method |
| AP | anti-personnel (mine) |
| ARL | Army Research Laboratory, Adelphi, Maryland, USA |
| AT | anti-tank (mine) |
| BiCG | biconjugate gradient method |
| BiCGStab | biconjugate gradient stabilized method |
| BOR | body of revolution |
| CEM | computational electromagnetics |
| CFIE | combined field integral equation |
| CG | conjugate gradient |
| CGN | conjugate gradient method operating on normal equation |
| CGS | conjugate gradient squared method |
| CPU | central processing unit |
| DCIT | discrete complex image technique (method of complex images) |
| DFT | discrete Fourier transform |
| EFIE | electric field integral equation |
| EM | electromagnetic |
| FD | frequency domain |
| FDFD | finite-difference frequency-domain method |
| FDTD | finite-difference time-domain method |

| | |
|---|---|
| FEM | finite element method |
| FFT | fast Fourier transform |
| FMM | fast multipole method |
| FOPEN | foliage penetrating (radar) |
| GPEN | ground penetrating (radar) |
| GPR | ground penetrating radar |
| GTD | geometrical theory of diffraction |
| HH | horizontally scattered and horizontally incident (plane wave) |
| HV | horizontally scattered and vertically incident (plane wave) |
| IE | integral equation |
| IFFT | inverse fast Fourier transform |
| ILM | impedance localization method |
| LU | in LU decomposition (lower and upper triangular matrices) |
| LUD | LU decomposition |
| MFIE | magnetic field integral equation |
| MLFMA | multi-level fast multipole algorithm |
| MoM | method of moments |
| MPIE | mixed potential integral equation |
| MRTD | multi-resolution time-domain method |
| PDE | partial differential equation |
| PEC | perfectly electric conducting (or perfect electric conductor) |
| PMC | perfectly magnetic conducting (or perfect magnetic conductor) |
| PVC | polyvinyl chloride |
| RAM | random access memory |
| RCS | radar cross section |
| RWG | triangular-patch basis introduced by Rao, Wilton, and Glisson [RWG82] |
| SAR | synthetic aperture radar |
| SGI | Silicon Graphics Indigo™ |
| SIE | surface integral equation |
| TD | time domain |
| TEM | transverse electromagnetic |
| $TE^{\zeta}$ | transverse electric with respect to $\zeta$-coordinate |
| TFQMR | transpose-free quasi-minimum residual method |
| $TM^{\zeta}$ | transverse magnetic with respect to $\zeta$-coordinate |
| UHF | ultra high frequency (300MHz–3GHz) |
| UTD | uniform geometrical theory of diffraction |
| UWB | ultra wideband |
| UXO | unexploded ordnance |

| | |
|---|---|
| VH | vertically scattered and horizontally incident (plane wave) |
| VHF | very high frequency (30MHz–300MHz) |
| VIE | volume integral equation |
| VV | vertically scattered and vertically incident (plane wave) |

## Lower Case Letters

| | |
|---|---|
| $a_\mu$ | complex amplitude coefficients used in DCIT (with $\mu=1,...,M$) |
| $b_\mu$ | complex exponent coefficients used in DCIT (with $\mu=1,...,M$) |
| $b_{\phi,t}^{k}(t')$ | subsectional basis (expansion) functions along generating arc in BOR MoM |
| $\mathbf{b}_n(\mathbf{x}')$ | vector basis (or expansion) functions in MoM (with $n=1,...,N$) |
| $c$ | speed of light $c=(\varepsilon\mu)^{-1/2}=c_0(\varepsilon_r\mu_r)^{-1/2}$ (only defined for lossless medium) |
| $c_0$ | speed of light $c_0=(\varepsilon_0\mu_0)^{-1/2}=2.997925\cdot 10^8$m/s in air (vacuum) |
| $d$ | length of source-observation vector $\mathbf{R}$ minus reference vector $\mathbf{X}$ in FMM |
| $d$ | group (cube, cluster) diameter in FMM |
| $d_\gamma$ | cube dimension (parallel to $x$-, $y$-, and $z$-direction) at level $\gamma$ in MLFMA |
| $d_j$ | thickness $d_j=z_{j-1}-z_j$ of layer $j$ in stratified medium (or section $j$ in transmission line) |
| $\mathbf{e}(t,\mathbf{x})$ | time-domain electric field vector |
| $f_c$ | center frequency of the spectrum of a time-domain waveform |
| $g$ | number of levels utilized in MLFMA |
| $g(\mathbf{x},\mathbf{x}')$ | scalar free-space Green's function $\exp(-jkR)/4\pi R$ with $R=\lvert\mathbf{x}-\mathbf{x}'\rvert$ |
| $g_h(\mathbf{x},\mathbf{x}')$ | scalar Green's function $\exp(-jk_hR)/4\pi R$ for the interior of homogeneous scatterer |
| $g_i(\mathbf{x},\mathbf{x}')$ | scalar Green's function $\exp(-jk_iR)/4\pi R$ for the direct radiation in layer $i$ |
| $g_h^{uv}$ | components of interior medium dyadic Green's function in BOR MoM |
| $\tilde{h}(k_\rho)$ | general spectral-domain function to be approximated in method of complex images |
| $h_n^\pm$ | projected heights on adjacent triangles forming RWG edge element $n$ |
| $i$ | index for source layer in stratified medium |
| $j$ | imaginary unit $j=\sqrt{-1}$ |
| $j$ | index to represent general layer in stratified medium |
| $k$ | wavenumber $k=\omega(\varepsilon\mu)^{1/2}$ |
| $k_0$ | wavenumber $k_0=\omega(\varepsilon_0\mu_0)^{1/2}=\omega/c_0$ of air (vacuum) |
| $k_{x,y,z}$ | components of wavenumber vector in Cartesian coordinates |
| $\bar{k}_{x,y,z1}$ | stationary phase point in near-to-far-zone transformation (wave vector in layer 1) |
| $k_{\rho,z}$ | radial and axial components of wavenumber vector in cylindrical coordinates |
| $\mathbf{k}$ | vector representing the direction of a plane wave with $\mathbf{k}=k\hat{\mathbf{k}}$ |
| $\hat{\mathbf{k}}$ | unit vector representing the direction of a plane wave |
| $l_n$ | length of common edge of adjacent triangles forming RWG edge element $n$ |
| $l_{1...3}^{q}$ | length of edge opposite to vertex $\mathbf{x}_{1...3}^{q}$ on source triangle $q$ (utilized in RWG basis) |
| $m$ | index for observation layer in stratified medium |

| | |
|---|---|
| $m$ | index for azimuthal Fourier mode in BOR MoM |
| $m$ | index for weighting function in triangular-patch MoM (not in FMM and MLFMA) |
| $m$ | index for observation group in FMM (in MLFMA denoted as $m_\gamma$) |
| $m'$ | index for source group in FMM (in MLFMA denoted as $m'_\gamma$) |
| $m_{\max}$ | upper limit for azimuthal Fourier modes $\lvert m\rvert\le m_{\max}$ needed in BOR MoM |
| $n$ | index for expansion function in triangular-patch MoM (not in FMM and MLFMA) |
| $n$ | index for weighting function in triangular-patch FMM and MLFMA |
| $n'$ | index for expansion function in triangular-patch FMM and MLFMA |
| $\hat{\mathbf{n}}$ | outward unit normal vector |
| $\hat{\mathbf{n}}_n^{\pm}$ | outward unit normal vectors on adjacent triangles forming RWG edge element $n$ |
| $p(t)$ | time-domain waveform characterizing an incident pulse |
| $p_{\theta,\phi}(t)$ | time-domain waveform characterizing an incident pulse for $\theta$- or $\phi$-polarization |
| $q(z,z')$ | determines phase function and image source location in method of complex images |
| $q_e$ | electric volume charge density (in As/m$^3$) |
| $q_{es}$ | electric surface charge density (in As/m$^2$) |
| $q_m$ | magnetic volume charge density (in Vs/m$^3$) |
| $q_{ms}$ | magnetic surface charge density (in Vs/m$^2$) |
| $\mathbf{r}'^{\pm}_n$ | local vectors $\mathbf{r}'^{\pm}_n=\pm(\mathbf{x}'-\mathbf{x}^{\pm}_{n0})$ used in definition of RWG expansion function $n$ |
| $\mathbf{r}^{c\pm}_n$ | local vectors $\mathbf{r}^{c\pm}_n=\pm(\mathbf{x}^{c\pm}_n-\mathbf{x}^{\pm}_{n0})$ for centroids on triangles of RWG edge element $n$ |
| $\mathbf{r}'^{q}_{1\ldots3}$ | local vectors $\mathbf{r}'^{q}_{1\ldots3}=\mathbf{x}'-\mathbf{x}^{q}_{1\ldots3}$ on source triangle $q$ (utilized in RWG basis) |
| $s_{pq}$ | components of time-domain polarimetric far-field scattering matrix $[\mathbf{s}]$ |
| $[\mathbf{s}]$ | time-domain polarimetric far-field scattering matrix (for infinite bandwidth) |
| $t$ | time variable |
| $t$ | coordinate along generating arc in BOR MoM |
| $\hat{\mathbf{t}}$ | unit vector arbitrarily tangential to a surface $S$ |
| $\hat{\mathbf{t}}$ | unit vector into the direction of the generating arc in BOR MoM |
| $u$ | coordinate for general curvilinear (locally) orthogonal coordinate system |
| $\hat{\mathbf{u}}$ | unit vector in curvilinear orthogonal $(u,v)$-system, tangential to a surface $S$ |
| $\hat{\mathbf{u}}$ | unit vector $\hat{\mathbf{u}}=-\hat{\phi}$ utilized in derivation of layered-medium Green's functions |
| $v$ | coordinate for general curvilinear (locally) orthogonal coordinate system |
| $\hat{\mathbf{v}}$ | unit vector in curvilinear orthogonal $(u,v)$-system, tangential to a surface $S$ |
| $\hat{\mathbf{v}}$ | unit vector $\hat{\mathbf{v}}=\hat{\rho}$ utilized in derivation of layered-medium Green's functions |
| $w_\kappa$ | weighting coefficients for numerical Gauss quadrature integration |
| $w_{pq}$ | components of normalized time-domain response matrix $[\mathbf{w}]$ |
| $[\mathbf{w}]$ | normalized time-domain response matrix for characterization of UWB scattering |
| $\mathbf{w}_m(\mathbf{x})$ | vector weighting (or testing) functions in MoM with (with $m=1,\ldots,M$) |
| $w^{l}_{\phi,t}(t)$ | subsectional weighting (testing) functions along generating arc in BOR MoM |
| $w^{ml}_{\phi,t}(\phi,t)$ | two-dimensional weighting (testing) functions applied in BOR MoM |

| | |
|---|---|
| $\mathbf{x}$ | general observation point $\mathbf{x}=x\hat{\mathbf{x}}+y\hat{\mathbf{y}}+z\hat{\mathbf{z}}$ |
| $\mathbf{x}'$ | general source point $\mathbf{x}'=x'\hat{\mathbf{x}}+y'\hat{\mathbf{y}}+z'\hat{\mathbf{z}}$ |
| $\mathbf{x}'_I$ | location of complex image (in DCIT) or real image (in FMM and MLFMA) |
| $\mathbf{x}_m$ | observation group center in FMM (in MLFMA denoted as $\mathbf{x}_{m_\gamma}$) |
| $\mathbf{x}_{m'}$ | source group center in FMM (in MLFMA denoted as $\mathbf{x}_{m'_\gamma}$) |
| $\mathbf{x}^I_{m'}$ | image source group center in FMM (in MLFMA denoted as $\mathbf{x}^I_{m'_\gamma}$) |
| $\mathbf{x}_{n1,n2}$ | vertices on common edge of adjacent triangles forming RWG edge element *n* |
| $\mathbf{x}^{\pm}_{n0}$ | distinct vertices of adjacent triangles forming RWG edge element *n* |
| $\mathbf{x}^{c\pm}_{n}$ | centroids of adjacent triangles forming RWG edge element *n* |
| $\mathbf{x}^{p,q}_{c}$ | centroids on observation (*p*) and source triangle (*q*) (utilized in RWG basis) |
| $\mathbf{x}^{p,q}_{1...3}$ | vertices on observation (*p*) and source triangle (*q*) (utilized in RWG basis) |
| $z^*$ | special coordinate defined for (asymptotic) near-to-far-zone transformation |
| $z_j$ | interface coordinates in layered-medium background (with $j=1,2,...,N$) |

## Capital Letters

| | |
|---|---|
| $\mathbf{A}$ | magnetic vector potential |
| $[\mathbf{A}]$ | general complex matrix used in the solution of a system of linear equations |
| $[\mathbf{A}']$ | equivalent real matrix used in the solution of a system of linear equations |
| $\mathbf{B}$ | magnetic flux density (in Vs/m$^2$) |
| $\mathbf{B}_{m'\alpha'}$ | spatial Fourier transform (radiation pattern) of a basis function $\mathbf{b}_{n'}$ |
| $\mathbf{B}^I_{m'\alpha'}$ | spatial Fourier transform of an image basis function in FMM or MLFMA |
| $\overrightarrow{C}_i$ or $\overleftarrow{C}_i$ | complex amplitude of upward/downward propagating incident TE$^z$ wave in layer *i* |
| $\overrightarrow{D}_i$ or $\overleftarrow{D}_i$ | complex amplitude of upward/downward propagating incident TM$^z$ wave in layer *i* |
| $\mathbf{D}$ | electric flux density (in As/m$^2$) |
| $E_0$ | magnitude of downward propagating incident electric field in top layer($E_{\theta 0}$, $E_{\phi 0}$) |
| $\mathbf{E}$ | electric field (in V/m) |
| $\mathbf{E}^{inc}$ | incident electric field due to impressed sources |
| $\mathbf{E}^{scat}$ | scattered electric field due to induced sources |
| $\mathbf{F}$ | electric vector potential |
| $G^{\alpha\beta}_{mi}(z,z')$ | Green's function in (equivalent) transmission-line model of layered-medium background (with $\alpha=V$ or $\alpha=I$, and $\beta=V$ or $\beta=I$ representing the kind of source) |
| $G^{TX,\alpha\beta}_{mi}$ | same as $G^{\alpha\beta}_{mi}$, including superscript $TX=TE$ or $TX=TM$ to denote TE$^z$ or TM$^z$ mode |
| $\overleftrightarrow{\mathbf{G}}_{Ami}$ | layered-medium dyadic Green's function relating electric current sources $\mathbf{J}$ in source layer *i* and magnetic vector potential $\mathbf{A}$ in observation layer *m* |
| $\underline{\overleftrightarrow{\mathbf{G}}}_{Ami}$ | same as $\overleftrightarrow{\mathbf{G}}_{Ami}$, but representing only interface interactions (without direct radiation) |
| $\overleftrightarrow{\mathbf{G}}_{Fmi}$ | layered-medium dyadic Green's function relating magnetic current sources $\mathbf{M}$ in source layer *i* and electric vector potential $\mathbf{F}$ in observation layer *m* |
| $\underline{\overleftrightarrow{\mathbf{G}}}_{Fmi}$ | same as $\overleftrightarrow{\mathbf{G}}_{Fmi}$, but representing only interface interactions (without direct radiation) |

| | |
|---|---|
| $\overleftrightarrow{\mathbf{G}}_{EJ,mi}$ | layered-medium dyadic Green's function relating source current $\mathbf{J}$ and field $\mathbf{E}$ |
| $\overleftrightarrow{\mathbf{G}}_{EM,mi}$ | layered-medium dyadic Green's function relating source current $\mathbf{M}$ and field $\mathbf{E}$ |
| $H_0$ | magnitude of downward propagating incident magnetic field in top layer($H_{\theta 0}$, $H_{\phi 0}$) |
| $\mathbf{H}$ | magnetic field (in A/m) |
| $I(z)$ | current used in (equivalent) transmission-line model |
| $I_0$ | localized current source in (equivalent) transmission-line model |
| $I_n$ | component of unknown vector $\mathbf{I}$ in MoM |
| $I_{n'}$ | component of unknown vector $\mathbf{I}$ in FMM and MLFMA |
| $I_{1,3}$ | scalar integrals used for (near) singularity extraction in triangular-patch MoM |
| $\mathbf{I}$ | vector of the unknown surface current coefficients in MoM |
| $\mathbf{I}^m$ | vector of the unknown current coefficients for Fourier mode $m$ in BOR MoM |
| $\mathbf{I}_{1,3}$ | vector integrals used for (near) singularity extraction in triangular-patch MoM |
| $J_{s\phi,st}^{mk}$ | unknown electric surface current coefficients (Fourier mode $m$) in BOR MoM |
| $\mathbf{J}$ | electric volume current density (in A/m$^2$) |
| $\mathbf{J}_s$ | electric surface current density (in A/m) |
| $K$ | number of plane waves $K$=2$L^2$ applied in FMM (in MLFMA denoted as $K_\gamma$) |
| $K$ | number of samples applied in discrete complex-image technique (DCIT) |
| $K_{\phi e}^{mi}$ | layered-medium scalar Green's function relating electric charge density $q_e$ in source layer $i$ and electric scalar potential $\phi_e$ in observation layer $m$ (for MPIE) |
| $\underline{K}_{\phi e}^{mi}$ | same as $K_{\phi e}^{mi}$, but representing only interface interactions (without direct radiation) |
| $K_{\phi m}^{mi}$ | layered-medium scalar Green's function relating magnetic charge density $q_m$ in source layer $i$ and magnetic scalar potential $\phi_m$ in observation layer $m$ (for MPIE) |
| $\underline{K}_{\phi m}^{mi}$ | same as $K_{\phi m}^{mi}$, but representing only interface interactions (without direct radiation) |
| $\overleftrightarrow{\mathbf{K}}_{Ami}$ | layered-medium dyadic Green's function in MPIE formulation (see also $\overleftrightarrow{\mathbf{G}}_{Ami}$) |
| $\underline{\overleftrightarrow{\mathbf{K}}}_{Ami}$ | same as $\overleftrightarrow{\mathbf{K}}_{Ami}$, but representing only interface interactions (without direct radiation) |
| $\overleftrightarrow{\mathbf{K}}_{Fmi}$ | layered-medium dyadic Green's function in MPIE formulation (see also $\overleftrightarrow{\mathbf{G}}_{Fmi}$) |
| $\underline{\overleftrightarrow{\mathbf{K}}}_{Fmi}$ | same as $\overleftrightarrow{\mathbf{K}}_{Fmi}$, but representing only interface interactions (without direct radiation) |
| $L$ | number of terms in FMM addition-theorem expansion (in MLFMA denoted as $L_\gamma$) |
| $\mathcal{L}_A$ | operator $\mathcal{L}_A=\mathcal{L}_{Ai}+\underline{\mathcal{L}}_A$ relating current density $\mathbf{J}$ to magnetic vector potential $\mathbf{A}$ |
| $\mathcal{L}_H$ | operator $\mathcal{L}_H=\mathcal{L}_{Hi}+\underline{\mathcal{L}}_H$ relating current density $\mathbf{J}$ to magnetic field $\mathbf{H}$ |
| $\mathcal{L}_\phi$ | operator $\mathcal{L}_\phi=\mathcal{L}_{\phi i}+\underline{\mathcal{L}}_\phi$ relating current density $\mathbf{J}$ to electric scalar potential $\phi$ |
| $\mathcal{L}_{EJ}^{int,ext}$ | operator relating electric current density $\mathbf{J}$ (source) to electric field $\mathbf{E}$ |
| $\mathcal{L}_{EM}^{int,ext}$ | operator relating magnetic current density $\mathbf{M}$ (source) to electric field $\mathbf{E}$ |
| $\mathcal{L}_{HJ}^{int,ext}$ | operator relating electric current density $\mathbf{J}$ (source) to magnetic field $\mathbf{H}$ |
| $\mathcal{L}_{HM}^{int,ext}$ | operator relating magnetic current density $\mathbf{M}$ (source) to magnetic field $\mathbf{H}$ |
| $M$ | number of groups (or clusters) in FMM (in MLFMA denoted as $M_\gamma$) |
| $M$ | number of complex exponentials applied in method of complex images |
| $M_{s\phi,st}^{mk}$ | unknown magnetic surface current coefficients (Fourier mode $m$) in BOR MoM |

| | |
|---|---|
| $\mathbf{M}$ | magnetic volume current density (in V/m$^2$) |
| $\mathbf{M}_s$ | magnetic surface current density (in V/m) |
| $[\mathbf{M}]$ | matrix used for preconditioning of a system of linear equations |
| $N$ | number of unknowns in (general and triangular-patch) method of moments |
| $N$ | number of subsections along the generating arc in BOR MoM |
| $N$ | number of interfaces in layered-medium background ($N$+1 layers) |
| $\tilde{N}_i^{TX,\alpha\beta}$ | used in the derivation of dyadic layered-medium Green's functions |
| $P$ | number of required iterations in conjugate-gradient solver |
| $P(\omega)$ | Fourier transform of an incident time-domain waveform $p(t)$ |
| $P_{\theta,\phi}(\omega)$ | Fourier transform of an incident time-domain waveform for $\theta$- or $\phi$-polarization |
| $P_{A,F}^{mi}$ | used in the derivation of dyadic layered-medium Green's functions |
| $Q_{A,F}^{mi}$ | used in the derivation of dyadic layered-medium Green's functions |
| $R$ | distance $R=\|\mathbf{x}'-\mathbf{x}\|$ between source point $\mathbf{x}'$ and observation point $\mathbf{x}$ |
| $R_q$ | distance between (quasi-static) real image source and observation point in DCIT |
| $R_\mu$ | complex "distance" between complex image source and observation point in DCIT |
| $\mathbf{R}$ | vector from source to observation point |
| $\overleftrightarrow{\mathbf{R}}$ | reflection dyadic utilized in (approximate) half-space FMM and MLFMA |
| $S$ | surface of a scatterer |
| $S^-$ | used to represent a surface slightly inside the surface $S$ |
| $S^+$ | used to represent a surface slightly outside the surface $S$ |
| $S_n^\pm$ | surface of adjacent triangles forming RWG edge element $n$ |
| $S_{m,n}$ | surface of RWG weighting ($m$) or basis ($n$) function in MoM |
| $S_{n,n'}$ | surface of RWG weighting ($n$) or basis ($n'$) function in FMM and MLFMA |
| $S^{p,q}$ | surface of observation ($p$) or source triangle ($q$) (utilized in RWG basis) |
| $S_m^\uparrow$ | surface located an infinitesimal distance outside surface $S_m$ (of a testing function) |
| $S_{pq}$ | components of polarimetric far-field scattering matrix $[\mathbf{S}]$ |
| $[\mathbf{S}]$ | polarimetric far-field scattering matrix |
| $T_L(\ldots)$ | translation operator utilized in FMM and MLFMA |
| $V$ | volume of a scatterer |
| $V(z)$ | voltage used in (equivalent) transmission-line model |
| $V_0$ | localized voltage source in (equivalent) transmission-line model |
| $V_m$ | component of driving vector $\mathbf{V}$ in MoM |
| $V_n$ | component of driving vector $\mathbf{V}$ in FMM and MLFMA |
| $\mathbf{V}$ | vector of the known excitation coefficients in MoM (driving vector) |
| $\mathbf{V}^m$ | vector of the known excitation coefficients for Fourier mode $m$ in BOR MoM |
| $\mathbf{W}_{m\alpha}$ | spatial Fourier transform (receiving pattern) of a weighting function $\mathbf{w}_n$ |
| $[\mathbf{W}]$ | sparse interpolation matrix used for interpolation and anterpolation in MLFMA |
| $X_{m'm}$ | length of vector between FMM group centers (in MLFMA denoted as $X_{m'_\gamma m_\gamma}$) |

| | |
|---|---|
| $X^I_{m'm}$ | length of vector between image and observation group centers (in MLFMA: $X^I_{m'_\gamma m_\gamma}$) |
| $\mathbf{X}$ | general complex vector used as unknown vector in a system of linear equations |
| $\mathbf{X}'$ | equivalent real vector used as unknown vector in a system of linear equations |
| $\mathbf{X}_{m'm}$ | vector between FMM source and observation group centers (in MLFMA: $\mathbf{X}_{m'_\gamma m_\gamma}$) |
| $\mathbf{X}^I_{m'm}$ | vector between image source and observation group centers (in MLFMA: $\mathbf{X}^I_{m'_\gamma m_\gamma}$) |
| $Y_0$ | characteristic admittance $Y_0 = 1/Z_0$ of (equivalent) transmission line |
| $\vec{Y}$ or $\overleftarrow{Y}$ | admittance in (equivalent) transmission-line model for looking into ±z-direction |
| $\mathbf{Y}$ | general complex right-hand side used in solution of a system of linear equations |
| $\mathbf{Y}'$ | equivalent real right-hand side used in solution of a system of linear equations |
| $Z_0$ | characteristic impedance of (equivalent) transmission line |
| $Z_{mn}$ | MoM impedance matrix elements in triangular-patch MoM |
| $Z_{nn'}$ | MoM impedance matrix elements in triangular-patch FMM or MLFMA |
| $\vec{Z}$ or $\overleftarrow{Z}$ | impedance in (equivalent) transmission-line model for looking into ±z-direction |
| $[\mathbf{Z}]$ | MoM impedance matrix |
| $[\mathbf{Z}^{\text{far}}]$ | MoM impedance matrix representing far interactions |
| $[\mathbf{Z}^{\text{far},i}]$ | MoM impedance matrix representing far direct radiation |
| $[\underline{\mathbf{Z}}^{\text{far}}]$ | MoM impedance matrix representing far-interface interactions |
| $[\mathbf{Z}^{\text{near}}]$ | MoM impedance matrix representing near interactions |
| $[\mathbf{Z}^m]$ | MoM impedance matrix for Fourier mode $m$ in BOR MoM |

## Greek Symbols

| | |
|---|---|
| $\alpha$ | angle $\alpha = \phi - \phi'$ between observation and source azimuth in BOR MoM |
| $\alpha$ | factor $0 \le \alpha \le 1$ in CFIE formulation for PEC scatterer |
| $\alpha$ | factor in PMCHW or Müller formulation for penetrable scatterer |
| $\alpha$ | index for weighting function within an observation group $m$ (in MLFMA: $\alpha_\gamma$) |
| $\alpha'$ | index for expansion function within a FMM source group $m'$ (in MLFMA: $\alpha'_\gamma$) |
| $\beta$ | factor in PMCHW or Müller formulation for penetrable scatterer |
| $\varepsilon$ | permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$ |
| $\varepsilon_0$ | permittivity $\varepsilon = \varepsilon_0 = 8.85419 \cdot 10^{-12}$As/Vm of air (vacuum) |
| $\varepsilon_r$ | relative complex permittivity $\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' - j\sigma/\omega\varepsilon_0$ (here we do not use $\varepsilon_r' + j\varepsilon_r''$) |
| $\phi_e$ | electric scalar potential |
| $\phi_m$ | magnetic scalar potential |
| $\gamma$ | index for the level in MLFMA |
| $\gamma$ | propagation constant $\gamma = jk_z$ of (equivalent) transmission line |
| $\gamma$ | angle of the generating arc relative to the $z$-axis in BOR MoM |
| $\vec{\Gamma}_j$ or $\overleftarrow{\Gamma}_j$ | reflection coefficient in (equivalent) transmission-line model for looking into ±z-direction, with the reference impedance chosen according to section (layer) $j$ |
| $\eta$ | wave impedance $\eta = (\mu/\varepsilon)^{1/2}$ |

| | |
|---|---|
| $\kappa$ | index for plane wave in FMM plane-wave expansion (in MLFMA denoted as $\kappa_\gamma$) |
| $\mu$ | permeability $\mu=\mu_0\mu_r$ |
| $\mu_0$ | permeability $\mu=\mu_0=4\pi\cdot10^{-7}$Vs/Am of air (vacuum) |
| $\mu_r$ | relative complex permeability $\mu_r=\mu_r'-j\mu_r''$ (here we do not use $\mu_r'+j\mu_r''$) |
| $\rho_{\max}$ | maximum radius of a body of revolution |
| $\sigma$ | electric conductivity (in S/m) |
| $\omega$ | circular (radian) frequency $\omega=2\pi f$ |
| $\omega_c$ | circular (radian) center frequency $\omega_c=2\pi f_c$ of a time-domain waveform spectrum |
| $\xi$ | radial coordinate used for spatial-domain Green's function (i.e., $\xi\cos\psi=x-x'$) |
| $\psi$ | azimuth coordinate used for spatial-domain Green's function (i.e., $\xi\cos\psi=x-x'$) |

## Subscripts and Superscripts

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|---|---|
| $\alpha$ | denotes weighting function within a FMM observation group $m$ (in MLFMA: $\alpha_\gamma$) |
| $\alpha'$ | denotes expansion function within a FMM source group $m'$ (in MLFMA: $\alpha'_\gamma$) |
| $\alpha\beta$ | denotes $\alpha=V,I$ (observed) and $\beta=V,I$ (source) in (equiv.) transmission-line model |
| $A$ | denotes fields due to (impressed or equivalent) electric current sources |
| $b$ | denotes quantities of the background medium (e.g., layered background) |
| $c$ | denotes centroid of a triangle |
| *equiv* | denotes equivalent sources (in volume or surface equivalence principle) |
| *ext* | denotes exterior field quantities (in volume or surface equivalence principle) |
| *EFIE* | denotes EFIE part of combined-field integral equation formulation |
| $F$ | denotes fields due to (impressed or equivalent) magnetic current sources |
| $\gamma$ | denotes the level in MLFMA |
| $h$ | denotes interior quantities for scattering from a homogeneous penetrable target |
| $i$ | denotes layer within a stratified medium in which source point resides |
| $ii$ | replaces $mi$ by $mi=ii$ if source and observation point are within the same layer $m=i$ |
| *inc* | denotes quantities related to the incident fields |
| *int* | denotes interior field quantities (in volume or surface equivalence principle) |
| $I$ | denotes image source or image source group in DCIT, FMM, and MLFMA |
| $j$ | general index to denote a layer within a stratified medium |
| $k$ | denotes subsectional basis functions along generating arc in BOR MoM |
| $\kappa$ | denotes plane wave in FMM plane-wave expansion (in MLFMA denoted as $\kappa_\gamma$) |
| $l$ | denotes subsectional weighting functions along generating arc in BOR MoM |
| $l$ | denotes $l$th term in addition-theorem expansion |
| $m$ | denotes layer within a stratified medium in which observation point resides |
| $m$ | denotes azimuthal Fourier mode in BOR MoM |
| $m$ | denotes weighting function in triangular-patch MoM (not in FMM and MLFMA) |
| $m$ | denotes observation group in FMM (in MLFMA denoted as $m_\gamma$) |

| | |
|---|---|
| $m'$ | denotes source group in FMM (in MLFMA denoted as $m'_\gamma$) |
| *MFIE* | denotes MFIE part of combined-field integral equation formulation |
| $n$ | denotes expansion function in triangular-patch MoM (not in FMM and MLFMA) |
| $n$ | denotes weighting function in triangular-patch FMM and MLFMA |
| $n'$ | denotes expansion function in triangular-patch FMM and MLFMA |
| $p$ | denotes observation triangle in triangular-patch MoM |
| $pq$ | polarization of scattered and incident wave, respectively ($pq$=$\theta\theta$, $\theta\phi$, $\phi\theta$, or $\phi\phi$) |
| $pq$ | polarization of scattered and incident wave, respectively ($pq$=VV, VH, HV, or HH) |
| $q$ | denotes source triangle in triangular-patch MoM |
| *scat* | denotes quantities related to the scattered fields |
| *TE* | denotes equivalent transmission-line quantities for $\mathrm{TE}^z$ waves |
| *TM* | denotes equivalent transmission-line quantities for $\mathrm{TM}^z$ waves |
| *TX* | denotes either $\mathrm{TE}^z$ waves (for *TX*=*TE*) or $\mathrm{TM}^z$ waves (for *TX*=*TM*) |
| $u,v$ | denote vector or dyadic components in locally orthogonal coordinate system |
| $x,y,z$ | denote vector or dyadic components in Cartesian coordinates |
| $\rho,\phi,z$ | denote vector or dyadic components in cylindrical coordinates |
| $r,\theta,\phi$ | denote vector or dyadic components in spherical coordinates |
| $\phi,t,n$ | denote vector components in conformal BOR coordinate system |
| $\pm$ | denotes whether observer is slightly outside ($S^+$) or inside ($S^-$) the surface $S$ |
| $\pm$ | denotes adjacent triangles $S_n^\pm$ forming RWG edge element $n$ |
| $\uparrow$ | denotes that an observation point is located slightly outside a surface $S$ |