

Anisotropic Inelasticity – Modelling, Simulation, Validation

D i s s e r t a t i o n

zur

Erlangung des akademischen Grades eines
Doktor-Ingenieurs (Dr.-Ing.)
des Fachbereiches Bauingenieurwesen und Geodäsie
der Technischen Universität Darmstadt

vorgelegt von

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Hauptreferent:	Prof. Dr.-Ing. habil. Friedrich Gruttmann
Korreferent:	Prof. Dr.-Ing. habil. Charalampos Tsakmakis
Tag der Einreichung:	05.07.2004
Tag der mündlichen Prüfung:	30.09.2004

Darmstadt

D 17

Berichte aus der Mechanik

Bernhard Eidel

**Anisotropic Inelasticity –
Modelling, Simulation, Validation**

D 17 (Diss. TU Darmstadt)

Shaker Verlag
Aachen 2005

Bibliographic information published by Die Deutsche Bibliothek

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the internet at <http://dnb.ddb.de>.

Zugl.: Darmstadt, Techn. Univ., Diss., 2004

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Printed in Germany.

ISBN 3-8322-4473-5

ISSN 1616-0126

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: www.shaker.de • eMail: info@shaker.de

Abstract

The main purpose of this work is threefold. First, to consider a phenomenological model for anisotropic elastoplasticity at finite plastic strains within the framework of continuum mechanics with internal variables. The model aims to describe metallic solids, where the source of anisotropy may be either a kind of texture in polycrystals, which allows for an orthotropic modelling in an averaged sense, or the anisotropy of fcc single crystals. The second thrust is to derive and implement within the finite-element framework stable and efficient time integration algorithms for the evolution equations of elastoplasticity, for both, finite plastic strains and for the infinitesimal strain case. The algorithmic formulation of unilateral contact in this work opens the door to the simulation of metal forming processes such as deep-drawing and extrusion. The third aim is to validate the material model along with its corresponding numerical treatment. Significant anisotropic phenomena of crystalline materials at large, inelastic deformations should be captured within the simulations. It is shown that the earing effect of a deep drawn cup is predicted. Furthermore, the present work is devoted to elucidate a curious phenomenon of a spherical indentation test into a fcc single crystal; upon release of the ball, the indent looks rather like a square than like a circle. The topography of the indentation crater is reconstructed applying scanning electron microscopy along with digital image processing. The anisotropic topography of the sample is predicted by the material model in the simulations. Corroborated on the experimental findings a kinematical explanation of the observed deformation pattern is proposed that operates on the micromechanical scale of plastic deformation in a fcc single crystal.

Kurzfassung

Das wesentliche Ziel dieser Arbeit ist dreifältig. Zunächst wird ein phänomenologisches Materialmodell für anisotrope Elastoplastizität bei grossen plastischen Deformationen im Rahmen der Kontinuumsmechanik mit internen Variablen vorgestellt. Das Modell zielt darauf ab, metallische Festkörper zu beschreiben, deren anisotropes Verhalten entweder von einer näherungsweise orthotropen Textur in Polykristallen verursacht ist, oder durch die Struktur eines kubisch-flächenzentrierten (kfz) Einkristalls. Das zweite Ziel ist, stabile und effiziente Zeitintegrationsverfahren für die plastischen Evolutionsgleichungen zu entwickeln, sowohl für grosse plastische Deformationen wie auch für den Fall kleiner Deformationen. Die vorgestellte Methode des unilateralen Kontakts im Rahmen der Finite-Element-Methode ermöglicht die Simulation typischer Umformprozesse wie Tiefziehen oder Extrusion. Das dritte Ziel ist, das Materialmodell samt der zugehörigen numerischen Verfahren zu validieren. Es gilt, signifikante anisotrope Materialphänomene, die bei grossen inelastischen Deformationen auftreten, in numerischen Berechnungen zu erfassen. Es wird gezeigt, dass die sog. Zipfelbildung am Rand eines tiefgezogenen Bechers vorhergesagt wird. Darüberhinaus widmet sich diese Arbeit einem verblüffenden Materialphänomen, das bei einem Eindrückversuch mit einer Kugel in einen kfz-Einkristall aufgetreten ist; der verbleibende Eindrückkrater gleicht eher einem Quadrat denn einem Kreis. Es gelingt in dieser Arbeit, die Topographie des Kraters mit Rasterelektronenmikroskopie und digitaler Bildverarbeitung zu rekonstruieren. Die anisotrope Topographie des Eindrucks wird durch das Materialmodell in Simulationen richtig vorhergesagt. Gestützt auf den experimentellen Befund wird eine physikalisch plausible Erklärung des Deformationsmusters vorgeschlagen, die auf diskreten Gleitvorgängen in der kfz-Elementarzelle gründet.

Acknowledgements

The work presented in this thesis was carried out during my stay at the Institute of Materials and Mechanics (IWMB) at Darmstadt University of Technology (TUD) between 1999 and 2004.

I thank Professor Dr.-Ing. habil. Friedrich Gruttmann to entrust me with the research project A15 "Finite element algorithms for engineering structures at finite deformations" within the special collaborative research program SFB 298 "Deformation and failure in metallic and granular materials". Leaving me the freedom and the responsibility for the directions and results in this and the follow-up project, I owe him valuable experiences due to this situation: from the cradle of fund-raising, over setting up and following the research priorities, to writing final reports about the results. Professor Gruttmann's kind generosity supported my work. The necessity of an up-to-date computational environment and the financial aid of his institute for the participation at conferences never had been a point of discussion. This support and his service as a referee is gratefully acknowledged.

I thank Professor Dr.-Ing. habil. Charalampos Tsakmakis for several discussions on plasticity and for a substantial comment on intermediate configurations. This helped to make an important point in my work clear and precise. Moreover, I thank Professor Tsakmakis for serving as a referee.

I thank Professor Dr.-Ing. M. Vormwald for some remarks on my work and for his service in the doctoral examination committee.

This work was mainly financed by the German Research Foundation (DFG). First, within SFB 298 for the 3-year period from 2000 till the end of 2002. Then, from may 2003 to may 2004, within the follow-up research project under grant GR 1136/3-1. Before, in October 1999, I attended a CISM-course on adaptive finite elements in Udine/Italy, equipped with a grant of the DFG. This support is gratefully acknowledged.

In the summer of 2003 I participated as a fellowship awardee at Professor Bathes "Second MIT-Conference on Computational Fluid and Solid Mechanics" in Cambridge/USA. This grant is gratefully acknowledged.

My thanks go to Dipl.-Ing. Volkmar Mehling and to Dipl.-Ing. Christian Fell, whom I had the pleasure to supervise, Volkmar in his Vertieferarbeit, Christian in his Diplomarbeit. We had many, highly rewarding discussions on mechanical and computational topics, and beyond.

I thank Dr.-Ing. Klaus Wintrich for some information in September 2003 about his former indentation experiment within SFB 298, project A12, which had shown a curious phenomenon. This phenomenon was not understood to that time. Dr. Wintrich then relinquished me the sample of the indentation experiment. This was my point of departure to elucidate this gripping problem.

I'm indebted to Alexander Gröber from **alicon** in Graz/Austria and to Wolfgang Joachimi from **point electronic** in Halle/Germany. The assistance in microscopy and image processing might not have been a big deal for them; for me, in a difficult situation, when conventional techniques continuously had failed and time ran short, their help in spring 2004 was a great joy.

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