

**Tadeus Uhl**

# **Performance Analysis of Queuing Systems**

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Berichte aus der Telematik

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**Performance Analysis of Queuing Systems**

Shaker Verlag  
Aachen 2015

**Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

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Printed in Germany.

ISBN 978-3-8440-3450-9

ISSN 0948-700X

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

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## Foreword

When packet-switching technology was implemented in networks towards the end of the 1960s the importance of queuing theory increased enormously. The underlying principle is simple: storage, processing and forwarding. That is how queuing systems work, not only in network technology but in transport and shipping as well. Queuing theory is a powerful and highly efficient tool that can be used to model, analyse and optimise all manner of processes in such systems. So it is well worth taking a closer look at this theory and putting it to good, practical use. This book will prove to be an important source of help, providing as it does reliable procedures for solving practical problems. It begins with a succinct presentation of the necessary basic principles of queuing theory. Liberal use of practical examples is used in favour of too much theory. The book focuses on the description of precise, numerical and simulative methods for observing queuing systems. Each chapter concludes with a series of questions and exercises that underline the practical use of the subject matter covered. The book concludes with special questions dealing with network technology and transportation and shipping; solutions are included. The book has been designed so that every student of telecommunications, applied informatics or comparable subjects can understand and learn the subject matter at hand. The author has drawn on many years of experience teaching at the Maritime University of Szczecin, Poland and the Flensburg University of Applied Sciences, Germany. The material can naturally be accompanied by relevant laboratory experiments to reinforce learning, and nothing prevents students applying the numerous modelling tools regularly used in queuing theory. The author wishes readers many new insights and many hours of enjoyment from this book.

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