

**Analysis and Applications
of Variational Sensitivity Information
in Structural Optimisation**

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Berichte aus der Mechanik

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Abstract

This thesis is concerned with the analysis of the *internal structure of sensitivities* of engineering structures with respect to modifications in shape. The term *internal structure of sensitivity* is introduced as an abbreviation for the eigenvalues and singular values, the corresponding eigenvalue spectrum and singular value spectrum as well as for the associated eigenvectors and singular vectors of the sensitivity matrix, the pseudo load matrix and the mesh velocity matrix, which build up the central parts of the sensitivity analysis. These matrices are analysed both qualitatively and quantitatively utilising the singular value decomposition (SVD) and techniques which are based on the principle component analysis (PCA).

The impact of the chosen models on the computed optimal designs, especially the influence of the chosen shape parametrisation, is analysed. This knowledge enables the design engineer to understand and improve the models systematically whereas they are usually set up entirely by engineering experience and intuition. The weaknesses of the models are detected and improved design descriptions are proposed. This human controlled process is called *design exploration*. The aim of this thesis is to contribute new substantial capabilities to the corresponding methods.

Moreover, an algorithmic and automatic treatment of SVD based sensitivity information is presented within this thesis for different kinds of application. In context of model reduction, the complete design space is reduced to the most valuable subspace of design modifications in order to demonstrate the information content of the decomposed sensitivities. Illustrative examples show that reasonable optimal designs can be obtained with a small percentage of properly defined design variables. In addition, the area of application for SVD based sensitivity information is extended to the nonlinear buckling analysis. Here, decomposition of the pseudo load matrix is utilised to generate the ‘worst case’ imperfections.

The generic concept is applied to shape optimisation of shell structures. The design of such structures is extremely important for their stability, robustness and load-bearing capacity. The variational design sensitivity analysis for a nonlinear solid shell is performed and especially the pseudo load matrix and the sensitivity matrix are derived. Within the scope of this thesis, only static nonlinear structural analysis and hyper-elastic material behaviour are considered.

Kurzfassung

Die vorliegende Arbeit befasst sich mit der *inneren Struktur der Empfindlichkeiten* von mechanischen Strukturen bezüglich geometrischer Veränderungen. Der Begriff *innere Struktur der Empfindlichkeiten* wird als abkürzende Bezeichnung für die Eigenwerte und Singulärwerte, die entsprechenden Eigenwert- und Singulärwertspektren, sowie die zugehörigen Eigenvektoren und singulären Vektoren der Pseudolast-, Sensitivitäts- und Designgeschwindigkeitsmatrizen eingeführt. Zusammen bilden diese Größen den Kern der Sensitivitätsanalyse und werden sowohl qualitativ als auch quantitativ mit Hilfe der Singulärwertzerlegung (SVD) und Techniken, die aus dem Bereich der Hauptkomponentenanalyse (PCA) bekannt sind, analysiert.

Beschrieben wird der Einfluss der Modellbildung, insbesondere die Wahl der Formparametrisierung auf die Lösung der Optimierungsaufgabe. Dieses Wissen ermöglicht es dem entwerfenden Ingenieur das Modell zu verstehen und es systematisch zu verbessern, was gewöhnlich nur auf seiner Erfahrung und Intuition basiert. Die Schwächen der Modellbildung werden identifiziert und verbesserte Parametrisierungen des Designraumes vorgeschlagen. Ein solches Vorgehen, das unter anderem die Interaktion zwischen Mensch und Maschine erfordert, wird auch als *Designexploration* bezeichnet und stellt den Schwerpunkt der vorliegenden Arbeit dar.

Des Weiteren wird eine algorithmische und automatische Behandlung der auf SVD basierten Sensitivitätsinformationen für verschiedene Anwendungen vorgestellt. Im Zusammenhang mit der Modellreduktion wird der vollständige Designraum auf einen Unterraum mit der größtmöglichen Varianz projiziert, um den Informationsgehalt der Sensitivitätszerlegungen zu demonstrieren. Beispiele werden zeigen, dass nur ein Bruchteil der neu definierten Designvariablen benötigt wird, um brauchbare Optimierungsergebnisse zu erzielen. Das Anwendungsgebiet der SVD basierten Sensitivitätsinformationen wird auf die nichtlineare Beulanalyse ausgeweitet. Hierbei werden die singulären Vektoren der Pseudolastmatrix mit den 'worst case' Imperfektionen in Verbindung gebracht.

Die entwickelten Konzepte werden auf die Formoptimierung von Schalenträgwerken angewandt. Das Design solcher Strukturen hat einen großen Einfluss auf ihre Stabilität, Robustheit und ihre Versagenslast. Die variationelle Sensitivitätsanalyse einer nichtlinearen Schale wird durchgeführt. Insbesondere werden die Sensitivitäts- und Pseudolastmatrizen hergeleitet. Es werden nur statische Probleme mit hyperelastischem, auch nichtlinearem, Materialverhalten betrachtet.

Preface

The work presented in this thesis was carried out between 2008 and 2013 being a research assistant at the chair of Numerical Methods and Information Processing at TU Dortmund. The financial support of the German Research Foundation (DFG) under grant number BA 1828/3-1 and BA 1828/5-1 is gratefully acknowledged.

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