

# **Optimization-based Solutions to Constrained Trajectory-tracking and Path-following Problems**

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