Atmospheric correction formulae for space geodetic techniques

Dudy D. Wijaya

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Abstract

The International Association of Geodesy (IAG) has recognized the need of global long-term Earth observations and consequently has inaugurated the Global Geodetic Observing System (GGOS) in July 2003. GGOS is dedicated to ensuring precise long-term monitoring of the geodetic observables, the dynamics of the atmosphere and ocean, the global hydrological cycle, as well as natural hazards and disasters, related to a global reference frame defined by integrating different geodetic techniques, models and approaches (www.ggos.org).

Within the framework of IAG's GGOS service, measurements using three different space geodetic positioning systems, i.e. Satellite Laser Ranging (SLR), Global Navigation Satellite Systems (GNSS), and Very Long Baseline Interferometry (VLBI), must be carried out at the millimeter level of accuracy. Furthermore, the measurement of the main geodetic observables using these techniques should be accomplished in a consistent manner to ensure a world-wide homogeneous and long-term interpretation of the measurement results. However, the demand to increase the accuracy of the measurements has revealed that their ultimate accuracy is limited by the hardware systems and atmospheric propagation effects. In addition, different atmospheric correction models applied for these space geodetic techniques might jeopardize the consistency of the atmospheric range corrections.

The research presented in this thesis mainly deals with the corrections for the atmospheric propagation effects of space geodetic techniques. In order to ensure the consistency of the atmospheric range corrections, a new concept of atmospheric correction modeling is proposed, with all developments of atmospheric corrections being based on this concept. The developments conducted in this research consist of three distinct sections: (1) atmospheric correction formula for two-frequency SLR measurements, (2) unified atmospheric corrections for space geodetic techniques, and (3) corrections of the atmospheric propagation effects using co-located observations of different space geodetic techniques.

In the first section, a new atmospheric correction formula for two-frequency SLR measurements was developed. This new formula properly eliminates the total atmospheric density effects and takes into account all of the remaining propagation effects, except for those caused by atmospheric turbulence. The water vapor distribution and curvature (*arc-to-chord*) effects are properly modeled in the new formula. Numerical simulations show that this new formula significantly reduces all propagation effects at any elevation angle to an accuracy better than 1 mm. It therefore has a much better performance than the existing formula. However, the required precision for the difference of the two-frequency SLR measurements, i.e. better than 8 μm, exceeds the capability of current state-of-the art SLR systems. An averaging technique is thus proposed to improve the precision of the measurements.

The required information about the water vapor distribution along the propagation path can be inferred using GPS or Water Vapor Radiometer data. The accuracy demand on this data is moderate, thus the use of a co-located GPS receiver is proposed. Moreover, the *arc-to-chord* correction can be calculated by a moderately accurate model.

In the second section, a unified atmospheric correction formula was developed, which can be applied to correct for the propagation effects of the electrically neutral atmosphere and ionosphere on GNSS/VLBI measurements, as well as the effects of the electrically neutral atmosphere on SLR measurements. In the unified formula, all integrations are carried out along the known chord of the propagation path, gradients of the horizontal density are incorporated into the integrations, and the chord elevation angle is used instead of the unknown apparent elevation angle. The curvature effects through the electrically neutral atmosphere and the ionosphere are evaluated separately from the propagation delays. It has been shown that the unified formula can be used to develop a new model for the accurate calculation of the zenith propagation delays as well as a unified mapping function for all space geodetic techniques.

In the third section, which is based on the results of the previous developments, methods of utilizing co-located observations of SLR and GNSS techniques are proposed, which are useful to estimate the remaining water vapor effects of SLR measurements. There are two possible co-location scenarios: (i) co-located SLR and GNSS observations to a GNSS satellite equipped with a retroreflector, and

(ii) co-located SLR and GNSS observations to different satellites. For the first scenario, the Slant Wet Delay (SWD) can be calculated from a single GNSS signal (e.g. GPS-36, GPS-35, GLONASS-95, GIOVE-A and GIOVE-B satellites). The propagation paths of the SLR and GNSS signals (optical and microwave paths) deviate only slightly and hence they are assumed to carry the same information about the water vapor distribution. For the second scenario, the SWD values along the propagation paths of the SLR signal can be calculated by interpolating the SWD values obtained from processing the GNSS data.

Zusammenfassung

Die Internationale Assoziation für Geodäsie (IAG) erkannte die Notwendigkeit von globalen Langzeit-Erdbeobachtungen und hat infolgedessen im Juli 2003 das Globale Geodätische Beobachtungs System (GGOS) ins Leben gerufen. GGOS soll präzise Langzeitbeobachtungen von geodätischen Beobachtungsgrößen, der Dynamik der Atmosphäre und Ozeane und dem globalen hydrologischen Zyklus garantieren, ebenso wie jene von Naturgewalten und -katastrophen, jeweils in Bezug auf das globale Referenzsystem, welches durch Einbindung verschiedener geodätischer Methoden, verschiedener Modelle und Ansätze definiert wird (www.ggos.org).

Innerhalb des Netzwerkes von GGOS, müssen die Messungen der verwendeten drei geodätischen Positionierungssysteme [Satellite Laser Ranging (SLR), Global Navigation Satellite System (GNSS), und Very Long Baseline Interferometry (VLBI)] mit einer Genauigkeit im Millimeterbereich ausgeführt werden. Des weiteren sollen die Messungen der wichtigsten geodätischen Beobachtungsgrößen in einer konsistenten Art erfolgen, um eine weltweite homogene und langzeit Interpretation der Messergebnisse sicher zu stellen. Die Genauig- keitssteigerung der Messungen ist letztendlich durch die Hardware und die atmosphärischen Ausbreitungseffekte limitiert. Zusätzlich kann die Anwendung von verschiedenen Modellen zur atmosphärischen Korrektur für die verschiedenen Weltraumtechnologien die Konsistenz der Korrekturen der atmosphärischen Laufzeitverzögerungen gefährden.

Die in dieser Dissertation vorgestellte Forschungsarbeit beschäftigt sich hauptsächlich mit den Korrekturen der Effekte der atmosphärischen Laufzeitverzögerungen geodätischer Weltraumtechnologien. Um die Konsistenz der Laufzeitkorrekturen

zu garantieren, wird ein neues Konzept zur Modellierung der atmosphärischen Korrekturen vorgestellt, entsprechend erweitert und weiter entwickelt. Diese Entwicklungen können in drei verschiedene Abschnitte aufgeteilt werden: (1) Formel für die atmosphärischen Korrektur von Zweifrequenz SLR-Messungen, (2) Einheitliche atmosphärische Korrekturen für geodätische Weltraumtechnologien, (3) Korrektur der atmosphärischen Ausbreitungseffekte durch ortsgleiche Beobachtungen mittels verschiedener geodätischer Weltraumtechnologien.

Im ersten Abschnitt wird eine neue Formel für die atmosphärische Korrektur für Zweifrequenz SLR-Messungen abgeleitet. Diese neue Formel eliminiert den gesamten atmosphärischen Dichteeffekt und berücksichtigt alle verbleibenden Ausbreitungseffekte ausgenommen jene, die durch Turbulenzen in der Atmosphäre verursacht werden. Die Verteilung des Wasserdampfs und die Krümmungseffekte (Bogen zu Sehne) werden ausreichend genau modelliert. Numerische Simulationen zeigen, dass diese neue Formel alle Ausbreitungseffekte für jeden Elevationswinkel mit einer Geanuigkeit besser als 1mm eliminiert. Sie ergibt genauere Ergebnise als die existierende Methodik. Allerdings können die bestehenden SLR-Systeme die benötigte Messgenauigkeit von 8 μm nicht liefern. Für eine Verbesserung der Präzision der Messungen wird eine Mittelungsmethode vorgeschlagen.

Die benötigte Information über die Verteilung des Wasserdampfes entlang des Ausbreitungsweges kann durch GPS oder Wasserdampf-Radiometer Daten ermittelt werden. Die Genauigkeitsanforderungen an diese Daten ist moderat, weshalb die Verwendung eines ortsgleichen GPS-Empfängers vorgeschlagen wird. Die Korrektur für die Reduktion des Bogens zur Sehne kann durch ein einfaches Modell berechnet werden.

Im zweiten Abschnitt wird eine einheitliche Formel für die atmosphärische Korrektur der Ausbreitungseffekte der elektrisch neutralen Atmosphäre und Ionospäre bei GNSS/VLBI Messungen entwickelt. Auch die Effekte der elektrisch neutralen Atmosphäre bei SLR Messungen können damit reduziert werden. In dieser einheitlichen Formel für die atmosphärische Korrektur sind alle Integrationen entlang der bekannten Sehne des Ausbreitungsweges ausgeführt, die Gradienten der horizontalen Dichte werden in die Integration eingebunden und anstelle des nicht bekannten scheinbaren Elevationswinkels wird der bekannte Elevationswinkel der Sehne verwendet. Die Krümmungseffekte der elektrisch neutralen Atmosphäre und Ionospäre werden getrennt ausgewertet. Es wird gezeigt, dass diese einheitliche Formel für die Entwicklung eines neuen Modells für die genaue

Berechnung von Zenit-Laufzeitverzögerungen als auch für einheitliche *Mapping Functions* aller geodätischen Weltraumtechnologien verwendet werden kann.

Im dritten Abschnitt werden, aufbauend auf die vorhergegangenen Ergebnisse, ortsgleiche GPS- und SLR-Beobachtungstechniken vorgeschlagen, welche für die Schätzung des Einflusses des Wasserdampfes von Nutzen sind. Es existieren zwei mögliche Anordnungen: (i) ortsgleiche GPS- und SLR-Beobachtungen zu einem mit Reflektoren ausgestatteten GNSS-Satelliten und (ii) ortsgleiche GPS- und SLR-Beobachtungen zu verschiedenen Satelliten. Für die erste Anordnung kann der feuchte Anteil der Laufzeitverzögerung SWD (Slant Wet Delay) mittels eines GNSS-Signals berechnet werden (e.g. GPS-36, GPS-35, GLONASS-95, GIOVE-A and GIOVE-B). Der Ausbreitungsweg der SLR- und GNSS-Signale weicht nur leicht (optische und Mikro-wellenausbreitung) voneinander ab, weshalb der selbe Informationsgehalt bezüglich der Wasserdampfverteilung angenommen werden kann. Für die zweite Anordnung können die SWD-Werte entlang des Ausbreitungsweges des SLR-Signals durch Interpolation der SWD-Werte entlang des GNSS-Signals berechnet werden.

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When I completed my master's studies I felt that my theoretical knowledge on Geodesy was weak, and I have been therefore interested in carrying out theoretical research in the field of Satellite Geodesy. Fortunately, not long after having received my master degree, there was an opportunity to undertake PhD studies in Austria, supported by the Austrian South-East Asian University Partnership Network (ASEA-UNINET). I immediately contacted Prof. Fritz K. Brunner, asking him for a possibility to do theoretical research under his supervision, and I was very lucky as he supported my interest and accepted me as his student. Three and a half years of studying under his supervision was an invaluable experience for me, during which I gained a lot of guidance, constant support and encouragement from him. His constructive criticism, suggestions and inspiration greatly improved my knowledge and nurtured my skills. I profoundly respect and admire him for everything he has given to me.

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Mathematical symbols

Wave propagation

ψ	Wave function
k	Wave number
ω	Angular frequency
ω_o	Resonance frequency
f	Frequency
λ	Wavelength
Υ	Reciprocal of the wavelength, $\frac{1}{\lambda}$
$\mathcal{S}$	Wavefront
A	Amplitude
v	Phase velocity
v_g	Group velocity
c	The velocity of light
$\mathbf{E}$	Electric field
$\mathbf{B}$	Magnetic field
$\mathbf{J}$	Electric current density
$\mathbf{M}$	Magnetic polarization
$\mathbf{H}$	Magnetic field intensity
q_e	Electric charge
m_e	Mass of an electron
ρ_{el}	Electrical charge density
μ	Magnetic permeability

μ_o	Magnetic permeability of free space
μ_r	Relative permeability
ε	Electric permittivity
ε_o	Electric permittivity of free space
ε_r	Relative permittivity
$\mathbf{P}$	Dielectric polarization
α	Induced dipole moment
α_p	Orientation polarization
p_o	Permanent dipole moment
k_B	The Boltzmann's constant
γ	Damping factor
t	Time
τ	Relaxation time required for external field-induced of the molecules
$\boldsymbol{i}$	Imaginary part

Atmospheric parameters

ρ	The density of a molecule
ρ_t	Total density of the electrically neutral atmosphere
ρ_d	Dry density of the electrically neutral atmosphere
ρ_v	Water vapor density of the electrically neutral atmosphere
$\mathrm{N_e}$	Free electron density of the ionosphere
P	Total pressure
P_s	Total surface pressure
e	Water vapor pressure
e_s	Surface water vapor pressure
T	Temperature
T_s	Surface temperature
T_d	Dew-point temperature
T_m	Mean temperature
$\mathrm{N_m}$	Peak electron density of the ionosphere
$\tilde{r}$	The number of molecules per unit volume
R	Specific gas constant
R_d	Specific gas constant for dry air
R_v	Specific gas constant for water vapor

n	Phase refractive index
n_g	Group refractive index
n^m, n^o	Phase refractive indices for microwave and optical waves
N	Phase refractivity
N_g	Group refractivity
N^m, N^o	Phase refractivities for microwave and optical waves
N^{trp}, N^{ion}	Phase refractivities for the troposphere and ionosphere

k_1, k_2, k_3, k_5	Microwave refractivity constants
$k_d(\lambda)$, $k_v(\lambda)$	Dry/water vapor dispersion constants for the phase optical refractivity
$k_{dg}(\lambda)$, $k_{vg}(\lambda)$	Dry/water vapor dispersion constants for the group optical refractivity
$f(\lambda)$	Dispersion factor

Atmospheric turbulence parameters

l_o	The inner scale of turbulence
R_{fz}	The radius of the first Fresnel zone
L	Propagation distance
σ_α	The mean square angle-of-arrival fluctuation
σ_s	The wavefront fluctuation due to turbulence

Geometry of observations

$\vec{r}$	The vector of position
no, es, up	Topocentric coordinate
X, Y, Z	Geocentric coordinate
ξ	The running coordinate

ϱ_j	The radius of curvature along the j axis ($j = X, Y, Z$)
r_o	The radius of Earth
H	Height above the ground station
h_s	Height of a satellite
h_m	Height of the peak electron density

θ_i, θ_t	The incident and transmitted angles
ϵ_o	The apparent elevation angle
e_o	The chord elevation angle
ϕ	The radius angle
β	The vertical angle

$\mathcal{R}$	The optical path length
S	The chord distance
ds	The arc length
G	The geometric distance
κ	The curvature effect
P_{21}	The propagation correction from the path p_2 to p_1
P_{ic}	The propagation correction from the path p_i to the chord p_c

Atmospheric correction terms

τ_a	Atmospheric propagation effects
τ_a^z	Zenith atmospheric total delay
τ_h	Slant hydrostatic delay
τ_h^z	Zenith hydrostatic total delay
τ_v	Slant water vapor delay
τ_v^z	Zenith water vapor delay
τ_i	Slant ionospheric delay
τ_i^z	Zenith ionospheric delay

$\tau_{v_{\mathrm{g}}}$	Slant water vapor delay of GNSS measurements
$\tau_{v_{\mathrm{g}}}^{int}$	Interpolated slant water vapor delay of GNSS measurements
$\tau_{v_{2f}}$	Slant water vapor delay of two-frequency SLR measurements
$\tau_{v_{1f}}$	Slant water vapor delay of single-frequency SLR measurements

ν	The power of dispersion
H_{21}	The water vapor factor

P_{ic}^{trp}	The perturbation term of the electrically neutral atmosphere
P_{ic}^{ion}	The perturbation term of the ionosphere
P_{ic-trp}	The propagation correction due to the electrically neutral atmosphere
P_{ic-ion}	The propagation correction due to the ionosphere
P_{ic-mix}	The propagation correction due to the combined effects of the electrically neutral atmosphere and ionosphere
$\chi(e_o)_{trp}$	Scaling factor for a practical model of P_{ic-trp}
$\chi(e_o)_{ion}$	Scaling factor for a practical model of P_{ic-ion}
$\chi(e_o)_{mix}$	Scaling factor for a practical model of P_{ic-mix}

Mathematical operators

∇	Gradient operator
$\nabla\times$	The Curl operator
$\frac{\partial}{\partial t}$	Differential operator with respect to time t
$\int$	Integral operator
Δ	Difference operator
$\vert\vert$	Absolute operator

Stochastic parameters

σ	Standard deviation
σ^2	Variance
$\mathcal{B}$	Bandwidth of the noise
$\mathcal{L}_x$	Optimum record length, for which stationarity of a random variable must be assured

Acronyms

IAG	International Association of Geodesy
GGOS	Global Geodetic Observing System
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
VLBI	Very Long Baseline Interferometry
SLR	Satellite Laser Ranging
WMO	World Meteorological Organization
WVR	Water Vapor Radiometer
NWP	Numerical Weather Prediction
SIWV	Slant Integrated Water Vapor
SWD	Slant Wet Delay
ZTD	Zenith Total Delay
ZHD	Zenith Hydrostatic Delay
ZWD	Zenith Wet Delay
PWV	Precipitable Water Vapor
TEC	Total Electron Content
VTEC	Vertical Total Electron Content
EM	Electromagnetic
UV	Ultra Violet
OPL	Optical Path Length
KHz	Kilo Hertz
GHz	Giga Hertz
THz	Terra Hertz

RRE	Residual Range Error