

“A Branch-and-Price Framework for Workforce Scheduling Problems”

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zur Erlangung des Grades eines Doktors der Wirtschaftswissenschaften

vorgelegt von

Christoph Stark  
aus Wuppertal

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der Technischen Universität Clausthal

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Vorsitzende der Promotionskommission: Prof. Dr. Heike Y. Schenk-Mathes

Hauptberichterstatter: Prof. Dr. Jürgen Zimmermann

Berichterstatter: Prof. Dr. Christoph Schwindt

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**Christoph Stark**

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