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*The introduction of suitable abstraction is
our only mental aid to organize and master
complexity.*

EDSGER WYBE DIJKSTRA (1930-2002)

Preface

In Germany, every year a philologist panel announces the “Unwort des Jahres”. The announcement cites an expression that should be banished from the German language.¹ The following economic term has been assigned to be the

Unwort des Jahres 2004: Humankapital.

The Unwort Jury criticized - whether right or not - the diffusion of an economic technical term into other areas of the German language thereby indicating an increasing tendency to primarily judge all references to life in terms of economics. According to the jury, particularly politicians are subject to this tendency. I want to stress that it is not my purpose to challenge the jury’s expertise and provoke an academic dispute. But I want to take a stand.

I strongly believe in economics as a social science. It is close to the heart of each economist to understand the determinants of our lives and particularly to study the conditions that influence them. Even more I think that all references to life are touched by economics and therefore have to be judged as such - whether we like it or not. From my point of view complaining about this fact is a waste of resources. The opportunity costs of this are high: Discussions about socially and (or) politically preferable objectives are confusing and misleading. As a consequence, the desired

¹Permanent members: Prof. Dr. Margot Heinemann, Prof. Dr. Rudolf Hoberg, Dr. Nina Janich, Prof. Dr. Horst Dieter Schlosser; co-opted members in 2004: Dr. Friedrich Dieckmann, Volker Braun.

outcomes are harder to reach, if not missing entirely.

Apart from the chapter on the influence of paternal uncertainty on mating and child investment decisions, the main work of the dissertation at hand is not directly linked with understanding the determinants of our lives or with the conditions that influence them. Instead I try to contribute to the understanding of the toolboxes economists employ when they directly turn to these questions. My focus in doing so is on endogenous growth driven by human capital accumulation. Is this project disputable? Without foreclosing or interfering with the academic judgment of this dissertation, I stress that it is a worthwhile project - and I will extensively use the expression *human capital* throughout this dissertation.

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Notation

The notation follows the usual conventions, nevertheless the most important mathematical symbols that will be used are gathered in the first list. The economic meaning of the main parameters used in the subsequent six chapters is presented in the second list.

Mathematical symbols

$x \in X$	element
$[a, b]$	closed interval from a to b
(a, b)	open interval from a to b
$\mathbb{N}, \mathbb{N}_0, \mathbb{Z}$	$\{1, 2, \dots\}, \{0, 1, \dots\}, \{0, +1, -1, +2, -2, \dots\}$
$\mathbb{R}, \mathbb{R}_+, \mathbb{R}_{++}$	$(-\infty, +\infty), (0, +\infty), [0, +\infty)$
$f'(x)$	first derivative of function f with respect to x , i.e. $\frac{\partial f(x)}{\partial x}$
$f''(x), f^{(m)}(x)$	second, m -fold derivative of function f
$g_x(x, y), (g_y(x, y))$	first derivative of function g with respect to $x, (y)$
$g_{xx}(x, y), (g_{yy}(x, y))$	second derivative of function g with respect to $x, (y)$
\ln	natural logarithm
$\{x_i\}_{i=0}^n$	finite sequence
$\{x_i\}_{i=0}^\infty$	infinite sequence
$x_i \rightarrow x$	converges
$\sup, \max, \arg \max$	supremum, maximum, argument of the maximum operator

Economic parameters

Chapters 2 - 5

α	output elasticity of physical capital
β	(quarterly) subjective discount factor
γ	externality of human capital in goods production
ρ	(continuous) subjective discount rate
ρ_A	autoregressive parameter in state equation for total factor productivity
σ	inverse of the intertemporal elasticity of substitution
A	total factor productivity in goods production, a scale parameter
B	marginal (and average) productivity in the schooling sector

Chapter 6

α	weight of female contributions in child quality aggregation procedure
$1 - \alpha$	weight of male contributions in child quality aggregation procedure
δ	a man's probability of biological fatherhood
$1 - \delta$	paternal uncertainty
η	degree of substitutability between female and male contributions to child quality
γ	degree of parental altruism toward own offspring
σ	inverse of the intertemporal elasticity of substitution
κ	degree of concavity of utility with respect to quantity of own children
θ	degree of concavity of utility with respect to quality of own children
φ	sex ratio, i.e. the number of female individuals per male in society