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We study Monte Carlo methods for some classical problems of numerical analysis: the approximation of means, the approximation of integrals of smooth functions, and the approximate local solution of integral equations of the second kind with smooth kernel and right-hand side. We are interested in the complexity of these problems, that is, the cost (computing time) of optimal algorithms. This question is well studied for algorithms that can use random numbers uniformly distributed on $[0,1]$, which is the dominating model in the field of Monte Carlo methods (randomized algorithms). The cost of such optimal Monte Carlo methods is often much smaller than that of optimal deterministic methods.

In this work we are interested in the cost of optimal methods that use only random bits uniformly distributed on $\{0,1\}$. In particular, we want to minimize the used number of random bits. Moreover, we are interested in how the constants that appear in the complexity bounds depend on the dimension. Usually, the complexity bounds are for a fixed class of problem elements in a fixed dimension so that the dependence of the constants on the dimension is unknown.

We use the real number model of computation with an oracle for the computation of function values, and optimality is to be understood in the worst case sense. We first analyze the problem of the approximation of means. Building on the results for this problem we study the approximation of integrals and the solution of integral equations, that is, our approach to these two problems is to reduce them to the approximation of means.

We construct methods that have the optimal order of complexity, which is the same as that for Monte Carlo methods with random numbers from $[0,1]$. While known classical methods use about $d\,n$ of those random numbers with n denoting the number of function values and d the dimension, our methods for integration use only about $d \log n$, those for integral equations only about $d \log^2 n$ random bits. Concerning the problem of the integration of Hölder classes and that of the solution of integral equations on classes C^r we show upper bounds with constants explicitly depending on the dimension. For the Hölder classes and for the class C^1 the constants even grow only polynomially in the dimension.