

Bénard-Marangoni convection
at low Prandtl numbers -
Results of direct numerical simulations

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Thomas Boeck

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Abstract

Bénard-Marangoni convection has so far mainly been investigated for large Prandtl numbers. The thesis presents direct numerical simulations of this type of convection in low-Prandtl-number liquids (liquid metals), which explore strong thermocapillary forcing for the first time. Bénard-Marangoni convection occurs in a liquid layer with a free upper surface. For sufficiently strong heating from below, an instability mechanism based on the temperature dependence of surface tension amplifies surface tension gradients at the free surface. These gradients drive the flow by the Marangoni effect.

The thesis opens with a brief review of the literature and a detailed discussion of the mathematical model. Particular attention is paid to the appropriate limits of the governing partial differential equations for the case of vanishing Prandtl number. After that, the pseudospectral Fourier-Chebyshev scheme employed for the numerical simulations is presented. The parallelization strategy of the scheme and the parallel speedup are also discussed.

In the main part of the thesis, the results of the computer experiments are presented and analyzed. Two-dimensional convection is considered first. At the instability threshold, two-dimensional convection takes the form of steady rolls. Upon increasing the temperature difference applied across the layer it is found that the limit of vanishing Prandtl number becomes singular beyond a certain critical temperature difference. The singularity corresponds to an unlimited, approximately exponential growth of the flow amplitude with time. This so-called flywheel effect is also known from Rayleigh-Bénard convection. Finite Prandtl numbers remove the singularity. The flywheel effect then leads to a convective state called flywheel or inertial convection, in which the dissipation of mechanical energy is weak.

Further two-dimensional simulations explore the case of a free-slip bottom of the layer, which allows us to observe the asymptotic behavior of steady, laminar inertial convection. The results are in excellent agreement with a phenomenological boundary layer model. By contrast, with the more realistic no-slip condition, convection becomes turbulent for sufficiently strong heating. A phenomenological model is proposed to explain the turbulent scaling relations of integral flow quantities with the applied temperature difference.

Three-dimensional simulations show that the flow becomes quickly time-dependent upon increasing the applied temperature difference, and that inertial convection is unstable with respect to three-dimensional perturbations.

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