Robust Numerical Algorithms Based on Corrected Operator Splitting for Two-Phase Flow in Porous Media

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Vorgelegt von

Yufei Cao

aus Yantai, China

Hauptberichter:	Prof. Dr.rer.nat. Barbara Wohlmuth
Mitberichter:	Prof. Dr.-Ing. Rainer Helmig
	Prof. Dr.rer.nat. Ivar Aavatsmark
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Memorial for Prof. Magne S. Espedal

It was with great shock and sadness that I received the news that Prof. Magne S. Espedal passed away on January 25, 2010. Prof. Espedal was born in Norway in 1942, studied at the University of Bergen, Norway in 1962, and was employed at the Department of Mathematics, University of Bergen in 1971. In 1990, he was promoted to a full professor.

Prof. Espedal was crucial in establishing a research group and educational programs in reservoir mechanics in the Department of Mathematics. He was one of the main forces behind the creation of the interdisciplinary Center for Integrated Petroleum Research (CIPR) in Bergen, which is a national center of excellence. He also had many leading roles in Norwegian research, including the Norwegian Research Council. To many researchers, he was regarded as an informal leader of the Norwegian community in applied mathematics.

Prof. Espedal's scientific work covered a wide range of topics, and his research focus was the mathematical modeling of flow and transport processes in porous media. He had written several influential papers, often in close collaboration with graduate students and international colleagues on different subjects including operator splitting methods, domain decomposition methods and upscaling. Lately, he had been working on modeling microbial processes for enhanced oil recovery.

As one of my co-supervisors, I owe Prof. Espedal deep gratitude for his patient supervision. He kindly hosted me at CIPR, University of Bergen for three months and for another short visit afterwards. I am very grateful for his hospitality and constructive discussions on my work, especially the work done in Chapter 4 which is related to one of the research fields he had been working on. I would like to express my heartfelt condolence to his family. I dedicate this work to Prof. Espedal.

As a friendly mentor, I will miss him dearly.

Yufei Cao

Stuttgart, *April 2010*

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Big thanks to all my nice colleagues at the Institute of Applied Analysis and Numerical Simulation (IANS), particularly NMH, for creating a pleasant and harmonious working environment and providing many interesting non-scientific activities. Especially, I would like to thank Brit Steiner for her help on my administrative documents, thank Alexander Weiß for his nice discussions and work collaboration, thank Corinna Hager for her help on the German translation of the abstract of my work, and thank Annika Fuchs, a student from IANS, for sharing her code on the front-tracking method along streamlines.

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Yufei Cao

Stuttgart, April 2010

Contents

Notation

The following table shows the significant symbols used in this work. Local notations are explained in the text.

Symbol	Definition	Dimension
Greek Letters:		
α	Van Genuchten parameter	[Pa^{-1}]
Γ_D	Dirichlet boundary part of $\partial\Omega$	[-]
Γ_N	Neumann boundary part of $\partial\Omega$	[-]
γ	angle deformation	[rad]
	parameter needed in Van Genuchten relative permeability function	[-]
	angle between the streamline direction and the normal vector on a cell edge	[$^\circ$]
γ_{ij}	cell facet between cells K_i and K_j	[-]
Δt	time step size in the case without using operator splitting	[s]
	splitting step in the case using operator splitting	[s]
Δt_a	inner time step of the advection equation in operator splitting	[s]
Δt_d	inner time step of the diffusion equation in operator splitting	[s]
$\Delta x, \Delta y$	discretization lengths in x- and y-direction	[m]
ε	parameter needed in Van Genuchten relative permeability function	[-]
	dimensionless scaling factor	[-]
θ	contact angle	[$^\circ$]
λ	Brooks-Corey parameter (pore size distribution index)	[-]
λ_α	mobility of phase α	[(ms)/kg]
λ_t	total mobility	[(ms)/kg]

μ	dynamic viscosity	[kg/(ms)]
μ_α	dynamic viscosity of phase α	[kg/(ms)]
$\boldsymbol{\nu}$	scaled normal vector	[m]
ρ	density	[kg/m^3]
ρ_α	density of phase α	[kg/m^3]
σ	interfacial tension	[N/m^2]
τ	shear stress	[N/m^2]
	time-of-flight along a streamline	[s]
ϕ	porosity	[-]
ϕ_i	standard nodal basis function for node x_i of $\mathcal{T}_h$	[-]
χ	characteristic function	[-]
Ψ	slope limiter function	[-]
ψ_i	new basis function for interior node x_i of $\mathcal{T}_h$	[-]
Ω	solution domain	[-]
$\partial\Omega$	boundary of domain Ω	[-]

Latin Letters:

C, C_1, C_2	generic constants	[-]
$C(\Omega)$	linear space of continuous functions	[-]
$\mathcal{C}$	one-dimensional coarse grid	[-]
F_a	numerical advective flux	[m^2/s]
F_d	numerical diffusive flux	[m^2/s]
$\mathcal{F}$	one-dimensional fine grid	[-]
$\mathbf{G}$	gravity term	[kg/(m^{2}s^2)]
H	discretization length of the coarse grid $\mathcal{C}$	[m]
$H^{-s}(\Omega)$	dual space of $H^s(\Omega), s > 0$	[-]
$\mathcal{H}_h$	discrete solution operator of the diffusive step	[-]
$\mathbf{I}$	identity matrix	[-]
$\mathcal{I}_h, \hat{\mathcal{I}}_h, \mathcal{I}_h^*, P_h$	interpolation operators	[-]
$\hat{\mathcal{I}}_{\Gamma_N}, \mathcal{I}^*_{\Gamma_N}$	traces of $\hat{\mathcal{I}}_h, \mathcal{I}_h^*$ on Neumann boundary Γ_N	[-]
$\mathcal{J}$	partition of time interval	[-]
K	grid cell of MPFA mesh (or control volume for cell-centered finite volume method)	[-]
∂K	boundary of grid cell or control volume K	[-]
$\mathbf{K}$	intrinsic permeability	[m^2]
$\mathbf{K}_\alpha$	effective permeability of phase α	[m^2]
L	number of inner time steps $\triangle t_d$	[-]
$L^p(\Omega), H^s(\Omega)$	Sobolev spaces ($p = 2, \infty, s > 0$)	[-]
$\mathcal{L}$	space differential operator	[-]
$\mathcal{L}_a$	advection operator	[-]

$\mathcal{L}_d$	diffusion operator	[-]
N	number of time/splitting steps	[-]
$\mathcal{N}_h$	node index set of $\mathcal{T}_h$	[-]
P	global pressure	[Pa]
$\mathcal{P}_c^f, \hat{\mathcal{P}}_c^f$	prolongation operators from $\mathcal{C}$ to $\mathcal{F}$	[-]
$\mathbf{R}$	rotation matrix	[-]
$\mathcal{R}_f^c$	restriction operator from $\mathcal{F}$ to $\mathcal{C}$	[-]
$\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^d$	Euclidean spaces	[-]
ST	simplified L triangle	[-]
S_α	saturation of phase α	[-]
$S_{\alpha r}$	residual saturation of phase α	[-]
S_e	effective saturation	[-]
$\mathcal{S}_h$	discrete solution operator of the advective step	[-]
T	L triangle	[-]
T	simulation time	[s]
	temperature	[°C]
$\mathcal{T}_h$	finite element mesh	[-]
$\hat{\mathcal{T}}_h$	MPFA mesh (partition of domain Ω)	[-]
$\mathcal{T}_h^*$	dual mesh of $\mathcal{T}_h$	[-]
V	control volume for vertex-centered finite volume method	[-]
V_h	finite element space	[-]
V_h^*	dual volume element space	[-]
e	edge	[-]
	relative discrete L^2 norm of the error	[-]
$f, \bar{f}, \hat{f}, \tilde{f}$	fluxes through a certain edge	[m^2/s]
f_α	fractional flow function of phase α	[-]
g	(scalar) gravity	[m/s^2]
$\mathbf{g}$	gravity vector $(0, 0, -g)^T$	[m/s^2]
g_N	Neumann boundary value	[m/s]
h	maximum diameter of all grid cells	[m]
	discretization length of the fine grid $\mathcal{F}$	[m]
h_K	diameter of grid cell K	[m]
k	scalar intrinsic permeability	[m^2]
$k_{r\alpha}$	relative permeability of phase α	[-]
l	length of line segment	[m]
m	Van Genuchten parameter	[-]
n	Van Genuchten parameter	[-]
$\mathbf{n}$	unit normal vector	[-]
$p, \bar{p}$	pressure	[Pa]
p_α	pressure of phase α	[Pa]
p_c	capillary pressure	[Pa]

p_d	entry pressure	[Pa]
q	source/sink	[1/s]
q_α	source/sink of phase α	[1/s]
r	radius	[m]
	saturation gradient ratio	[-]
$\mathbf{s}$	streamline (nodes)	[-]
s	arc length along a streamline	[m]
t	time	[s]
	transmissibility coefficient	[$(m^3s)/kg$]
$\mathbf{t}$	unit tangential vector	[-]
u	unknown of a differential equation	[-]
$v^\perp$	normal velocity	[m/s]
$\mathbf{v}$	Darcy velocity	[m/s]
$\mathbf{v}_\alpha$	phase velocity of phase α	[m/s]
$\mathbf{v}_{\alpha m}$	modified phase velocity of phase α	[m/s]
$\mathbf{v}_a$	average velocity	[m/s]
$\mathbf{v}_t$	(Darcy) total velocity	[m/s]
$\mathbf{v}_{ta}$	average total velocity	[m/s]
$\mathbf{w}$	front velocity	[m/s]
$\mathbf{w}_a$	average front velocity	[m/s]
$x, \bar{x}, x', x'', y$	points in the Euclidean space	[-]
x_c	shock collision point	[-]

Subscripts:

α	phase, either wetting (w) or non-wetting (n)
K	grid cell
c	envelope
h	mesh size
n	non-wetting phase
res	residual flux
w	wetting phase

Superscripts:

D	Dirichlet boundary
L	left side
N	Neumann boundary
R	right side
d	number of dimensions
l	inner time step
n	time/splitting step